Soft Gluon Resummation in Dijet Production based on TMD Factorization

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LBNL

Outline

- QCD resummation based on TMD factorization
- QCD resummation for heavy quark pair production
- soft gluon resummation in dijet azimuthal angular correlation at hadron collider
- Summary

QCD k_T resummation

- Consider the production process $h_1 h_2 \rightarrow Z+X$

$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2}\{\ \alpha_S(L+1)\ \ +\alpha_S^2(L^3+L^2)\ \ +\alpha_S^3(L^5+L^4)+\alpha_S^4(L^7+L^6)+...$$
$$+\alpha_S^2(\ L+\ \ 1)\ \ +\alpha_S^3(L^3+L^2)+\alpha_S^4(L^5+L^4)+...$$
$$+\alpha_S^3(\ L+\ \ 1)+\alpha_S^4(L^3+L^2)+...\ \}$$

Where Q_T is the transverse momentum, and Q the mass of Z, and $L = \text{Log}[Q^2 / Q_T^2]$.

- We have to resum these large logs to make reliable predictions

$$W(Q,b) = e^{-\int_{1/b}^{Q}\frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C\otimes f_1 C\otimes f_2$$

TMD factorization

At the small transverse momentum limitation

$$W_{(\text{Drell-Yan})}=H(M,\mu)\, f_1(x_1,b,M,\mu)\, f_2(x_2,b,M,\mu)\, S(\lambda_T,\mu)$$

Fourier transformation ⇕ k_t

factorization scale

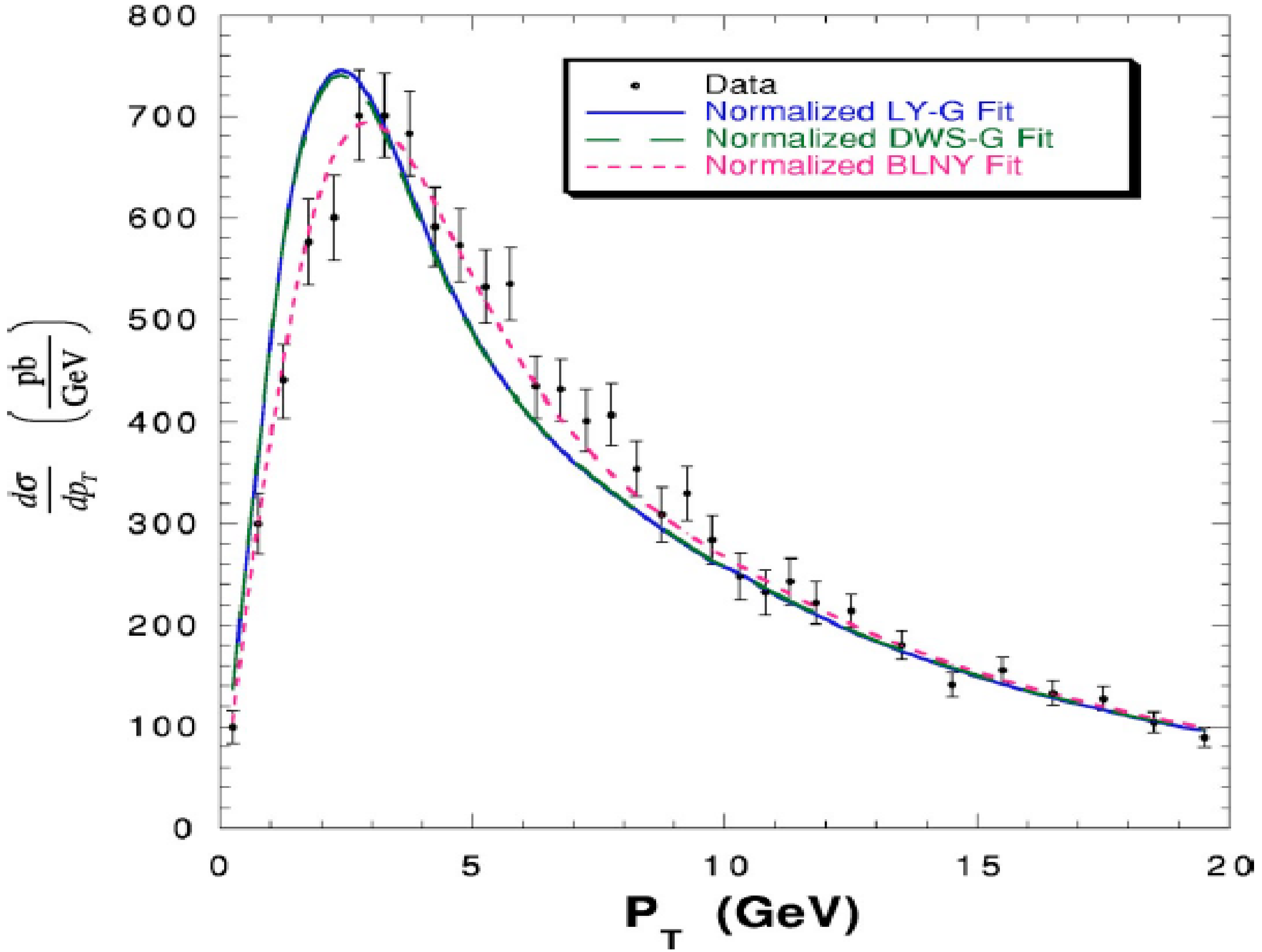
W satisfies CSS evolution equation

$$\frac{\partial W(x_i,b,M^2)}{\partial \ln M^2}=(K+G')W(x_i,b,M^2)$$

At one-loop order for Drell-Yan process

$$K(b,\mu)=-\frac{\alpha_s C_F}{\pi}\ln\frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu)=-\frac{\alpha_s C_F}{\pi}\left(\ln\frac{Q^2}{\mu^2}-\frac{3}{2}\right)$$

CDF Z Run 1



C. P. Yuan et al PRD 67, 073016

TMD factorization for heavy quark pair production

- We consider the process:

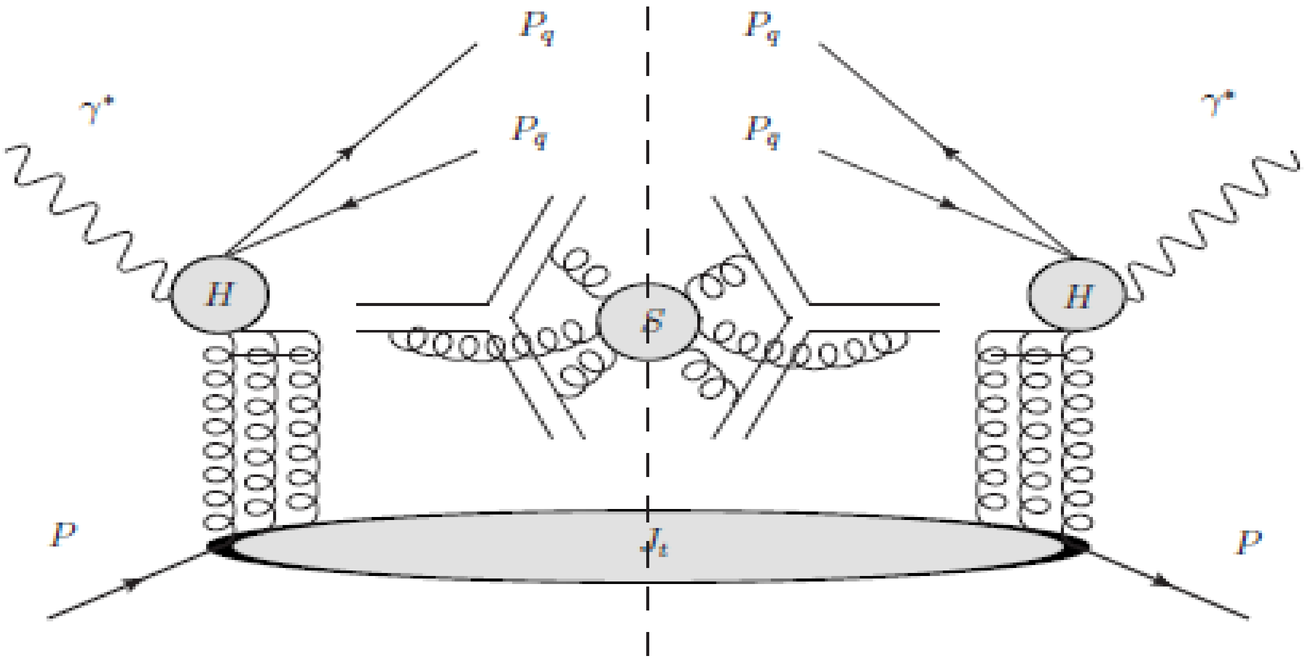
$$\gamma^* + p \to c\bar{c}[M_{c\bar{c}}, p_\perp] + X$$

There large logarithm log($M_{cc}/q_\perp$) at the small $q_\perp$ region.

- For this process, you can not apply the TMD factorization formulism of SIDIS directly.

- It is because the finial state is color-octet

R. Zhu, P. Sun and F. Yuan **Phys.Lett. B727 (2013)**

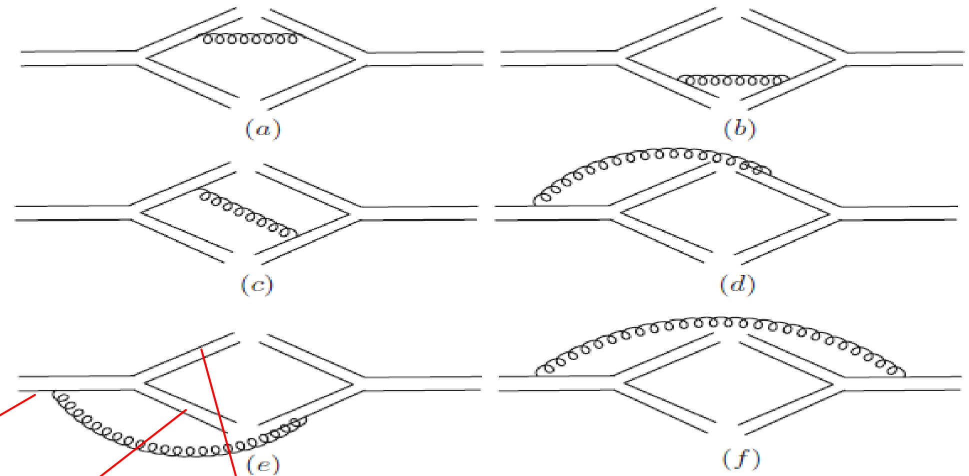


$$W_{c\bar{c}}(x, b_\perp) = H(\widetilde{Q}, \mu) x g(x, b_\perp, \widetilde{Q}, \mu) \overline{S}(b_\perp, \mu)$$

The TMD PDF still has the universality, but we should give a new definition of soft factor

And we do not need to make definition for heavy quark jet, because there is no light cone singularity in heavy quark jet.

The soft factor



The definition of soft factor:

$$\overline{S}(b_\perp,\mu,\rho) = \frac{\int_0^\pi \frac{(\sin\phi)^{-2\epsilon}}{a_1} d\phi \, \langle 0|\mathcal{L}^\dagger_{\bar{v}ca'}(b_\perp)\mathrm{Tr}\left[\mathcal{L}^\dagger_{n_c}(b_\perp)\mathrm{T}^{a'}\mathcal{L}^\dagger_{n_{\bar{c}}}(b_\perp)\mathcal{L}_{n_{\bar{c}}}(0)\mathrm{T}^{a}\mathcal{L}_{n_c}(0)\right]\mathcal{L}_{\bar{v}ac}(0)|0\rangle}{\mathrm{Tr}[\mathrm{T}^d\mathrm{T}^d]}$$

At the one loop order

$$\overline{S}^{(1)}_{\mathrm{JMY}}(b_\perp,\mu,\rho) = \frac{\alpha_s}{2\pi}\{C_A \ln\frac{c_0^2}{b_\perp^2\mu^2}\left(B_{final}+\ln\rho^2+\ln\frac{\tilde{Q}^2}{\zeta^2}-1\right)+C_{final}\}$$

Sudakov form factor:

$$S_{sud} = -\int_{\tilde{Q}_0}^{\tilde{Q}}\frac{d\mu}{\mu}\left(\ln\frac{\tilde{Q}}{\mu}\gamma_K(\mu)-\gamma_S(\mu,1)+\frac{\alpha_s C_A}{\pi}(1-2\beta_0-\ln\frac{\tilde{Q}_0^2 b_\perp^2}{c_0^2})\right)$$

$$\gamma_K(\mu) = \frac{2\alpha_s(\mu)C_A}{\pi},$$

$$\gamma_S(\mu,\rho) = -\frac{\alpha_s(\mu)C_A}{\pi}(B_{final}+\ln\rho-1)$$

- Then, we consider the process:

$$H_1(P_1) + H_2(P_2) \to c(k_1) + \bar{c}(k_2) + X$$

R. Zhu, C F Qiao, P. Sun and F. Yuan to be pubilshed

The W function will became

$$W_{kl}(x_i, b) = x_1 f_l(x_1, b, \xi_1^2, \mu^2, \rho) x_2 f_k(x_2, b, \xi_2^2, \mu^2, \rho) \mathrm{Tr}\left[\mathbf{H}(Q^2, \mu^2, \rho)\mathbf{S}(b, \mu^2, \rho)\right]$$

The Hard and soft part have to be expanded by a group of color basis

For the channel $q(i) + \bar{q}(j) \to t(k) + \bar{t}(l)$, we adopt the bases

$$C_1(i,j,k,l) = \delta_{ij}\delta_{kl}, \qquad C_2(i,j,k,l) = T^d_{ij}T^d_{kl}\,.$$

While for $g(a) + g(b) \to t(k) + \bar{t}(l)$, we have the bases

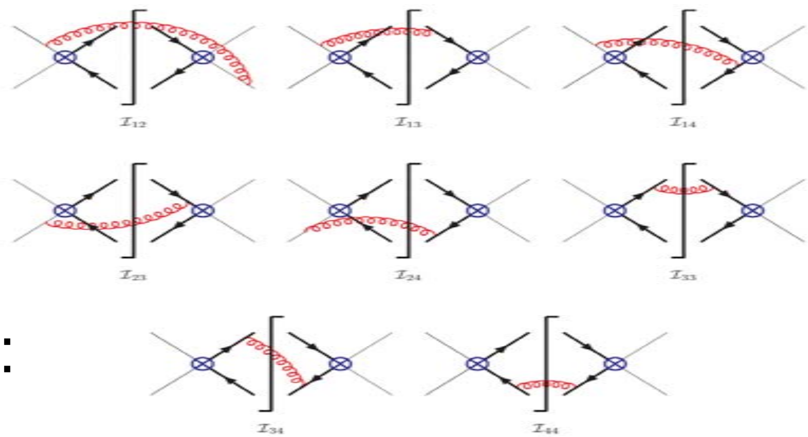
$$C_1(a,b,k,l) = \delta^{ab}\delta_{kl}, \qquad C_2(a,b,k,l) = if^{abd}T^d_{kl}, \qquad C_3(a,b,k,l) = d^{abd}T^d_{kl}$$

The soft factor's definition

We can definite the soft factor as:

$$S_{IJ} = \int_0^\pi \frac{(\sin\phi)^{-2\epsilon}}{\frac{\sqrt{\pi}\Gamma(\frac{1}{2}-\epsilon)}{\Gamma(1-\epsilon)}} d\phi \, C^{bb'}_{Iii'} C^{aa'}_{Jll'} \langle 0| \mathcal{L}^\dagger_{vcb'} \mathcal{L}_{vbc} \mathcal{L}^\dagger_{\bar{v}ca'} \mathcal{L}_{\bar{v}ac} \mathcal{L}^\dagger_{nji} \mathcal{L}_{ni'k} \mathcal{L}^\dagger_{nkl} \mathcal{L}_{nl'j} |0\rangle$$

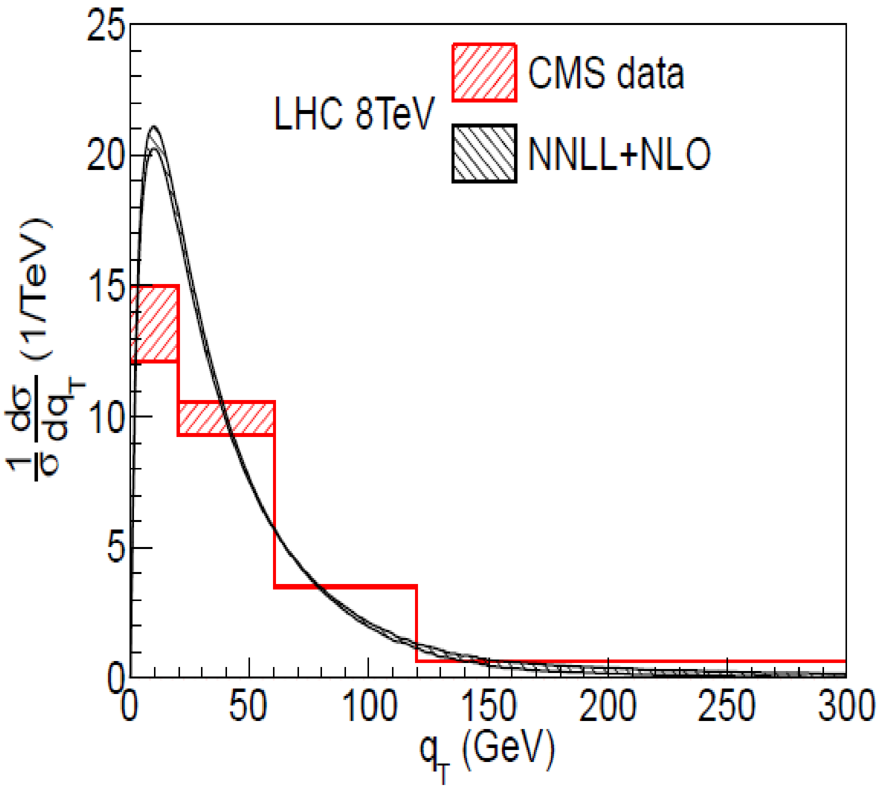
the evolution equation:

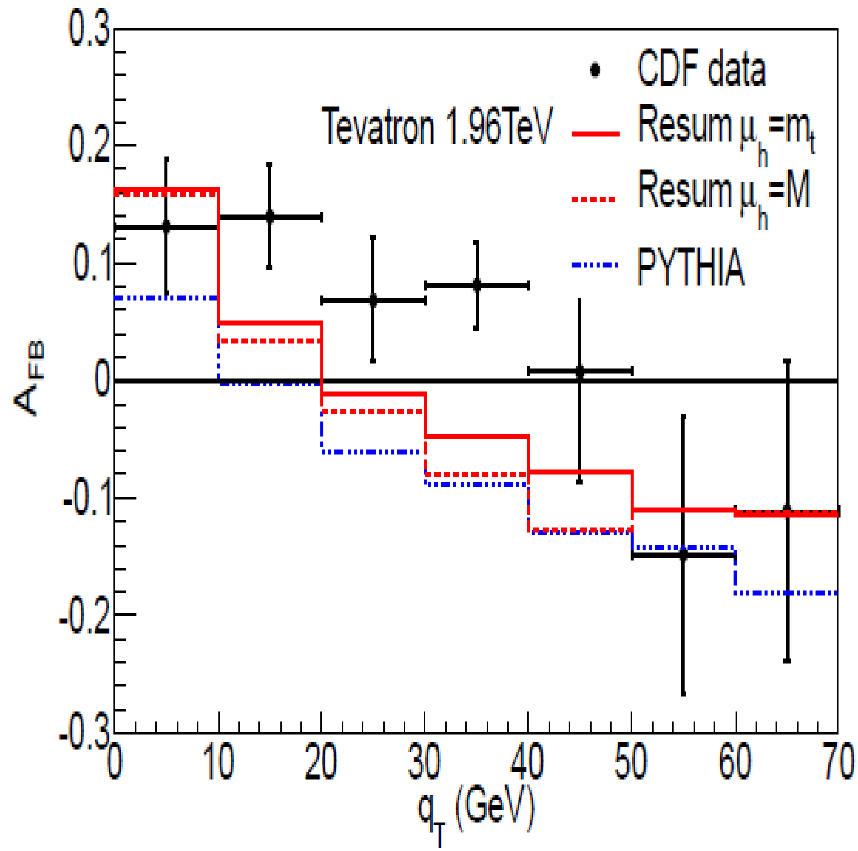


$$\frac{d}{d\ln\mu} S_{i\bar{i}}(\mu) = -\gamma^{s\dagger}_{i\bar{i}} S_{i\bar{i}}(\mu) - S_{i\bar{i}}(\mu)\gamma^{s}_{i\bar{i}}$$

Then you can get the W funtion:

$$W_{kl}\left(x_i, b_\perp, \frac{C_1^2}{C_2^2 b_\perp^2}\right) = f_k(x_A, C_1^2/(C_2^2/b_\perp^2)) f_l(x_B, C_1^2/(C_2^2/b_\perp^2))$$
$$\times Tr\left[\mathbf{H}(M^2_{c\bar{c}}, M^2_{c\bar{c}}) \mathrm{EXP}\{-\int_{C_1^2/b_\perp^2}^{M^2_{c\bar{c}}} \frac{d\mu}{\mu}\gamma^{s\dagger}_{i\bar{i}}\} \mathbf{S}(b, \frac{C_1^2}{C_2^2 b_\perp^2}) \mathrm{EXP}\{-\int_{C_1^2/b_\perp^2}^{M^2_{c\bar{c}}} \frac{d\mu}{\mu}\gamma^{s}_{i\bar{i}}\}\right]$$





These two pictures come from the paper:
C. S. Li et al Phys.Rev. D88 (2013) 074004
Where they used the SCET.
Our analytic result is consistent with theirs.

$$A_{FB}(q_T) = \frac{\sigma_F(q_T) - \sigma_B(q_T)}{\sigma_F(q_T) + \sigma_B(q_T)}$$

$$\sigma_F(q_T) = \int_0^1 d\cos\theta \, \frac{d^2\sigma}{d\cos\theta dq_T}$$

$$\sigma_B(q_T) = \int_{-1}^0 d\cos\theta \, \frac{d^2\sigma}{d\cos\theta dq_T}$$

Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

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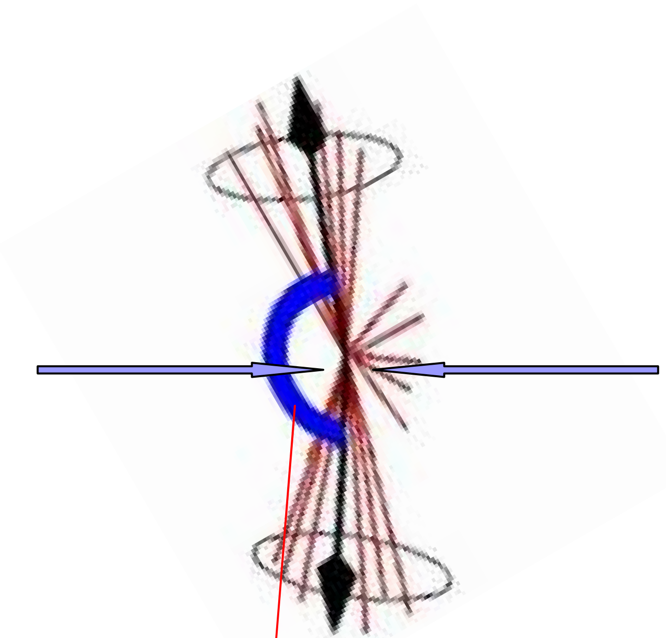
Abstract

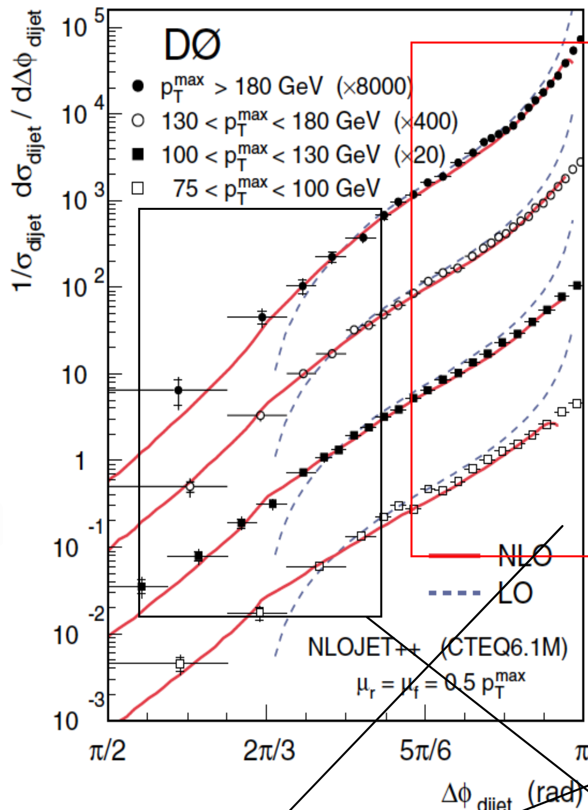
We derive the all order soft gluon resummation in dijet azimuthal angular correlation in *pp* collisions at the next-to-leading logarithmic level. The relevant coefficients for the resummation Sudakov factor, and the soft and hard factors are calculated. The theory predictions agree well with the experimental data from D0 collaboration at the Tevatron.

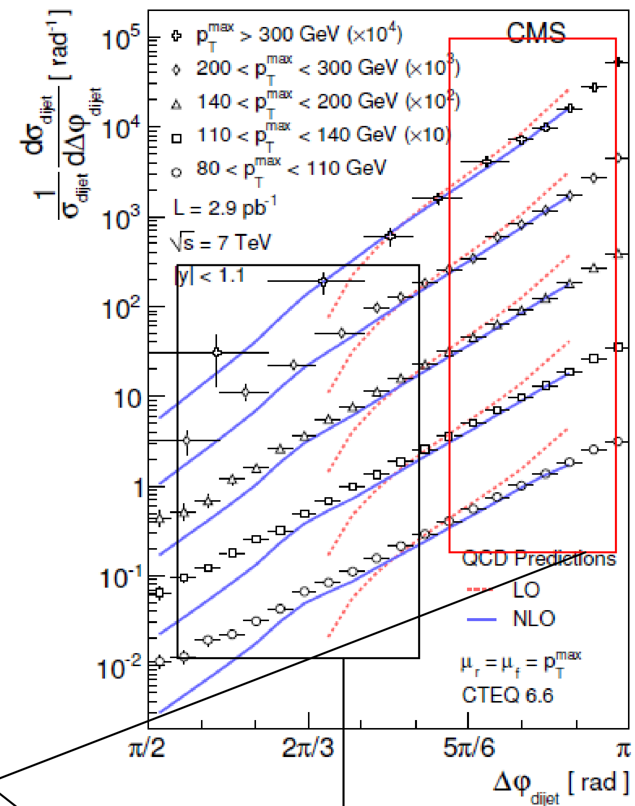
Motivations:

- reummation of large logs in dijet production
- factorization breaking effects
- Collins-Qiu, 2007; Vogelsang-Yuan, 2007; Rogers-Mulders 2010

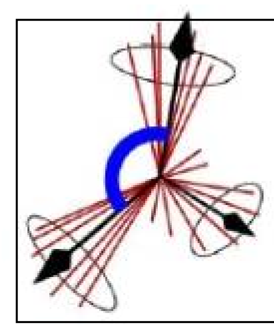


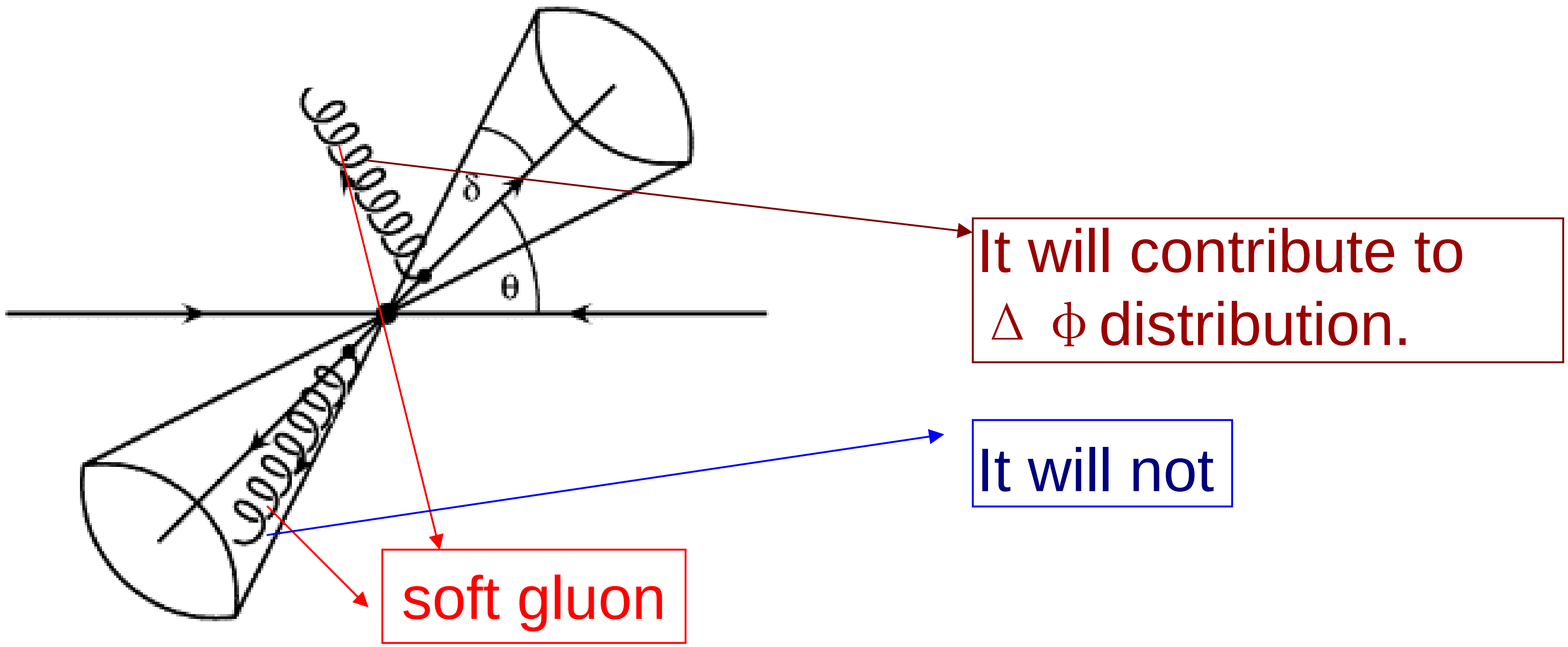
Δ φ





Here cross section is dominated by processes with only two jets.
The azimuthal angular distribution comes form soft gluon radiation.
QCD resummation is needed.





δ = R sin(θ), the R is the jet cone size, and its definition is:

$$R = (\Delta\phi^2 + \Delta y^2)^{1/2}$$

We take the small cone assumption.

So R and δ are small value.

So, you have to cut off the contribution form soft gluon in the jet cone. There are two ways to do this.

A

$$n_j = (1, 0 , 0 , 1) \rightarrow n_j^2 = 0$$

$$n_g = (1, 0 , \sin(x) , \cos(x)) \text{ with } x > \delta$$

$$n_j \cdot n_g = 1 - \cos(x) > \delta^2/2$$

B

$$n_j = (1^+, 0_\perp, \delta^2/2) \rightarrow n_j^2 = \delta^2$$

Then for any n_g ($n_g^2 = 0$) , we have :

$$n_j \cdot n_g > \delta^2/2$$

The difference between these two ways is proportional to δ .

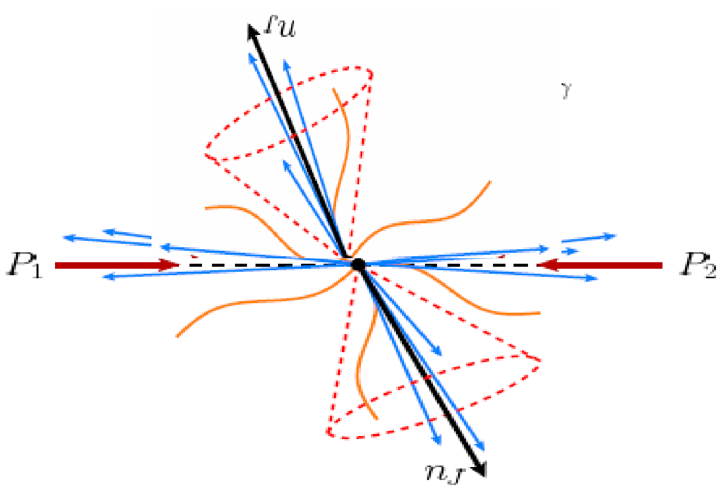
There three kinds of large logarithms in the processes: $(\mathrm{Log}(q_\perp/P_J))^2$, $\mathrm{Log}(q_\perp/P_J)$ and $\mathrm{Log}(R)\mathrm{Log}(q_\perp/P_J)$

$$\frac{d^4\sigma}{dy_1 dy_2 dP_J^2 d^2q_\perp} = \sum_{ab} \sigma_0 \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} W_{ab\to cd}(x_1, x_2, b_\perp) + Y_{ab\to cd} \right]$$

where

universal TMD PDF

$$W_{ab\to cd}(x_i, b) = x_1 f_a(x_1, b, \zeta^2, \mu^2, \rho) x_2 f_b(x_2, b, \bar{\zeta}^2, \mu^2, \rho) \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}(Q^2, \mu^2, \rho) \mathbf{S}_{ab\to cd}(b, \mu^2, \rho)\right]$$



As the same as heavy quark jets production, we do not need to make a definition for jets, because all the collinear gluons are covered by jet cone, and it will not contribute to $q_\perp$ distribution.

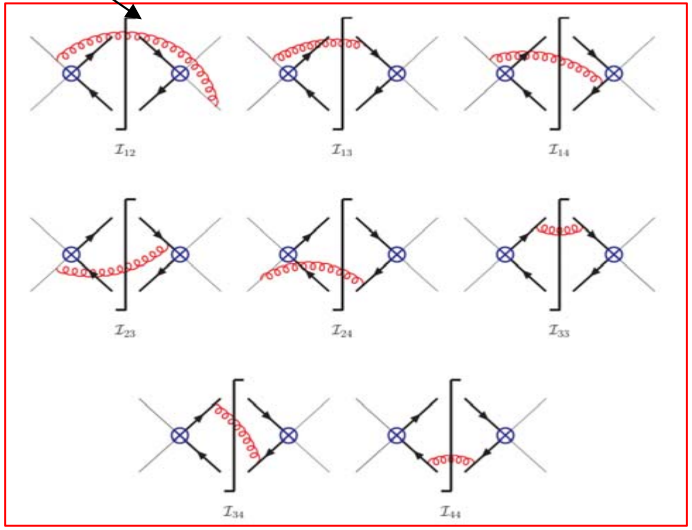
$$S_{IJ} = \int_0^{\pi} \frac{d\phi_0}{\pi} C^{bb'}_{Iii'} C^{aa'}_{Jll'} \langle 0|\mathcal{L}^{\dagger}_{vcb'}(b_\perp)\mathcal{L}_{vbc}(b_\perp)\mathcal{L}^{\dagger}_{\bar{v}ca'}(0)\mathcal{L}_{\bar{v}ac}(0)\mathcal{L}^{\dagger}_{nji}(b_\perp)\mathcal{L}_{\bar{n}i'k}(b_\perp)\mathcal{L}^{\dagger}_{\bar{n}kl}(0)\mathcal{L}_{nl'j}(0)|0\rangle$$

The soft factor satisfies

$$\frac{d}{d\ln\mu} S_{IJ}(\mu) = -\Gamma^{s\dagger}_{IJ'} S_{J'J}(\mu) - S_{IJ'}(\mu)\Gamma^{s}_{J'J}$$

$$c_1 = f^{a_1a_2c_1} f_{a_3a_4c_1}\,, \quad c_2 = f^{a_1a_3c_1} f_{a_2a_4c_1} + f^{a_1a_4c_1} f_{a_2a_3c_1}\,, \quad c_3 = d^{a_1a_2c_1} f_{a_3a_4c_1}\,,$$
$$c_4 = f^{a_1a_2c_1} d_{a_3a_4c_1}\,, \quad c_5 = d^{a_1a_4c_1} f_{a_2a_3c_1}\,, \quad c_6 = \delta^{a_1a_2}\delta^{a_3a_4}\,, \quad c_7 = \delta^{a_1a_3}\delta^{a_2a_4}\,, \quad c_8 = \delta^{a_1a_4}\delta^{a_2a_3}$$

$$c_1 = \delta^{a_1a_2}\delta_{a_3a_4}\,, \quad c_2 = i f^{a_1a_2c}\, t^c_{a_3a_4}\,, \quad c_3 = d^{a_1a_2c}\, t^c_{a_3a_4}$$



After solving the evolution equations

$$W_{ab\to cd}(x_1, x_2, b) = x_1 f_a(x_1, \mu = c_0/b_\perp) x_2 f_b(x_2, \mu = c_0/b_\perp) e^{-S_{Sud}(Q^2, b_\perp)} \times \mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{c_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{c_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}]\right]$$

where

$$S_{Sud}(Q^2, b_\perp, C_1, C_2) = \int_{C_1^2/b_\perp^2}^{C_2^2 Q^2} \frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right) A + B + D_1 \ln\frac{2}{R_1} + D_2 \ln\frac{2}{R_2}\right]$$

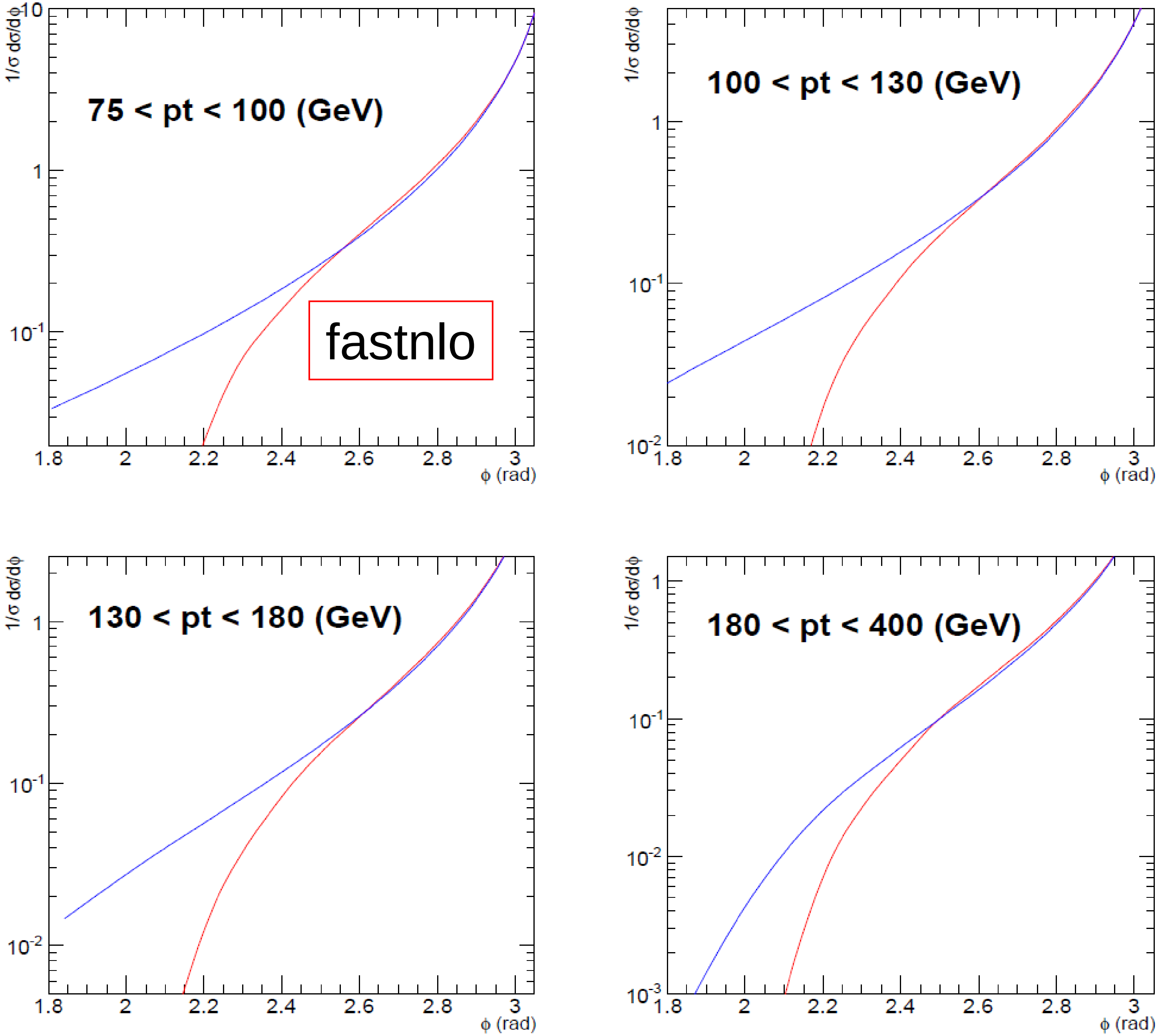
For g g → jj $A_{gg}=C_A\, a_s/\pi$ $B_{gg}=-2C_A\, \beta_0\, a_s/\pi$

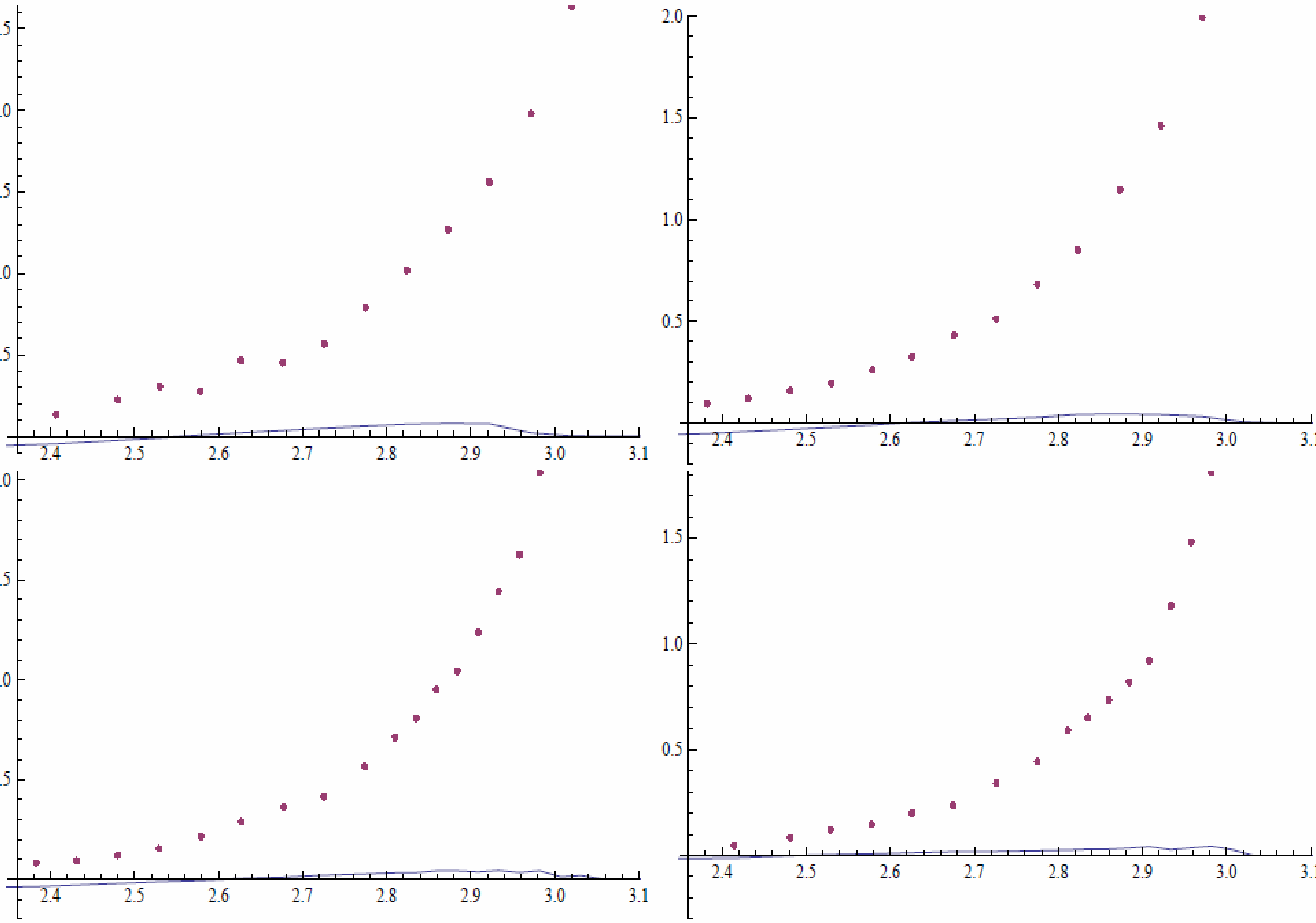
For q q → jj $A_{qq}=C_F\, a_s/\pi$ $B_{qq}=-2C_F/3\, a_s/\pi$

For q g → jj $A_{qg}=(A_{gg}+A_{qq})/2$ $B_{qg}= =(B_{gg}+B_{qq})/2$

for quark jet $D_i=C_F\, a_s/\pi$

for gluon jet $D_i=C_A\, a_s/\pi$


75 < pt < 100 (GeV)
fastnlo
100 < pt < 130 (GeV)
130 < pt < 180 (GeV)
180 < pt < 400 (GeV)
1/σ dσ/dϕ
ϕ (rad)
1.8
2
2.2
2.4
2.6
2.8
3
10
1
10^{-1}
10^{-2}
10^{-3}



2.5
2.0
1.5
1.0
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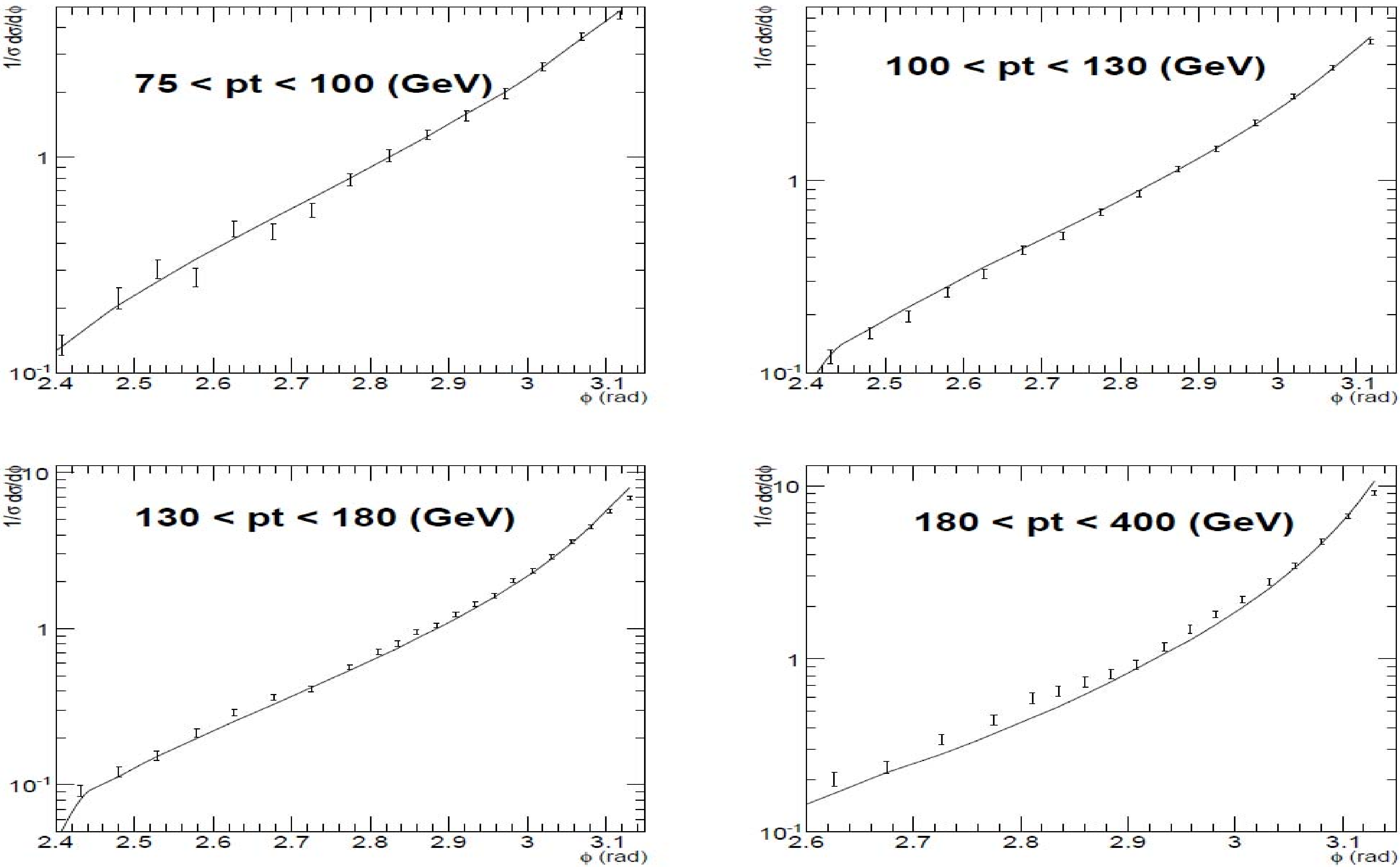
2.7
2.8
2.9
3.0
3.1



75 < pt < 100 (GeV)
100 < pt < 130 (GeV)
130 < pt < 180 (GeV)
180 < pt < 400 (GeV)
1/σ dσ/dφ
φ (rad)

Summary

- Our works shows that there is no problem to apply TMD factorization into dijet or heavy quark pair production processes at one-loop order.

- Based on the TMD factorization, QCD resummation works very well in phenomenology field for these processes.

- However, It is too hard for us to check our TMD factorization formalism at higher order of α_s.

Thank you very much!