Heavy Quark Energy Loss

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Suppression due to mass

Presence of mass generally suppress the likelihood of occurrence of a given scattering process

$$qq' \Rightarrow qq' \approx \frac{8}{9} g^4 \frac{s^2}{t^2}$$

$$qQ \Rightarrow qQ \approx \frac{8}{9} g^4 \frac{s^2}{t^2} \left(1 - \frac{M^2}{s}\right)^2$$

$$qq' \Rightarrow qq'g \approx 12 g^2 |\mathcal{M}_{2\to 2}|^2 \frac{1}{k_\perp^2}$$

$$qQ \Rightarrow qQg \approx 12 g^2 |\mathcal{M}_{2\to 2}|^2 \frac{1}{k_\perp^2} \underbrace{\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^2}_{\text{Dead Cone Factor}}$$

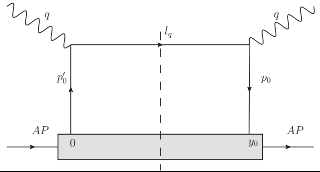
$$\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^2 \Rightarrow \left(1 + \frac{M^2}{s \tan^2 \frac{\theta}{2}}\right)^{-2}$$

$$\rightarrow \left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \quad \left(\text{when } \sqrt{s} \simeq 2E, \quad \tan\frac{\theta}{2} \simeq \frac{\theta}{2}\right)$$

Other mass effects?

Q. Are there any other aspects/effects qualitatively different to that of light quark (in context of jet quenching)?

heavy quark mass, modifications on momentum components



The incoming heavy quark momentum components :

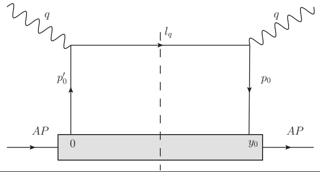
$$p \equiv \left[p^+, \frac{M^2}{2p^+}, 0, 0\right] = \left[x_B P^+, \frac{M^2}{2x_B P^+}, 0, 0\right]$$

Incoming virtual photon momenta components :

$$q = \left[-\frac{Q^2}{2q^-}, q^-, 0, 0\right]$$

Vanishing transverse components of the virtual photon are a choice of coordinate system, where as for the incoming quark vanishing transverse components are approximations.

mass modification on Bjorken variable (x_B)



Outgoing heavy quark has momentum components :

$$p + q = \left[x_B P^+ - \frac{Q^2}{2q^-}, \frac{M^2}{2x_B P^+} + q^-, 0, 0 \right]$$

There would be a shift in the parton distribution function caused by the new M dependent definition of the Bjorken variable x_B, given as

$$x_B \Rightarrow x_0 = \frac{1}{2} \frac{Q^2}{2P^+ q^-} \left(1 + \sqrt{1 + \frac{4M^2}{Q^2}} \right)$$

This is caused by the fact that the incoming quark has a negative light-cone momentum $p^- = M^2/(2p^+)$, *i.e.*, unlike a light quark, the heavy quark will have a non-vanishing p^-.

hadronic tensor ($W_0{}^{\mu\nu}$), light quark ($M \sim \lambda^2 Q$) *vs.* heavy quark ($M \sim Q$)

$$W_0^{A\,\mu\nu}{}_{M\to\lambda^2 Q} = c_p^A \; W_0^{\mu\nu}$$

$$= c_p^A \frac{2\pi}{2Q^2} \,\mathrm{Tr}\left[\not{p}\gamma^\mu \,(\not{p} + \not{q})\,\gamma^\nu\right] \sum_q Q_q^2 f_q(x_B)$$

$$= c_p^A \, 2\pi \left[\, -g^{\mu\perp} g^{\nu\perp}\right] \quad \sum_q Q_q^2 \int \frac{dy^-}{2\pi} e^{-ix_B p^+ y^-} \, \frac{1}{2} \langle p|\bar{\psi}(y^-)\gamma^+\psi(0)|p\rangle \,.$$

$$W_0^{A\,\mu\nu}{}_{M\to Q} = c_p^A \; W_0^{\mu\nu}{}_{M\to Q}$$

$$= c_p^A \frac{2\pi}{2Q^2} \frac{1}{\sqrt{1+4\frac{M^2}{Q^2}}} \,\mathrm{Tr}\left[(\not{p} + M)\,\gamma^\mu \;\; (\not{p} + \not{q} + M)\;\;\gamma^\nu\right] \; \sum_q \; Q_q^2 \; f_q(x_B)$$

$$= c_p^A \frac{2\pi}{2Q^2} \frac{1}{\sqrt{1+4\frac{M^2}{Q^2}}} \left[\, 8\; p^\mu p^\nu \; + 4 \left(p^\mu q^\nu \; + p^\nu q^\mu\right) \; - \; 4\; p.q \; g^{\mu\nu}\right] \; \sum_q \; Q_q^2 \; f_q(x_B)$$

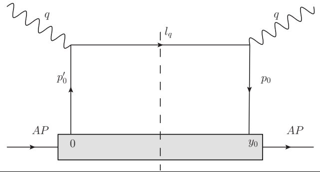
$$= c_p^A \, 2\pi \frac{1}{\sqrt{1+4\frac{M^2}{Q^2}}} \left[\, - g^{\mu\perp} g^{\nu\perp} \; + \; \frac{M^2}{(q^-)^2} g^{\mu -} g^{\nu -} \;\; + \ldots \right] \sum_q Q_q^2 \int \frac{dy^-}{2\pi} e^{-ix_B P^+ y^-} \, \frac{1}{2} \langle p|\bar{\psi}(y^-)\gamma^+\psi(0)|p\rangle \,.$$

$$W^{\perp\perp} \quad \sim \quad 1 \quad\quad 1$$

$$W^{+\perp} \sim W^{-\perp} \quad \sim \quad \lambda \quad\quad \lambda$$

$$W^{++} \sim W^{+-} \sim W^{--} \sim W^{-+} \quad \sim \quad \lambda^2 \quad\quad 1$$

how ‘W’ modified in medium?



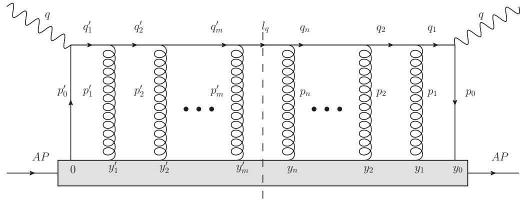
How the hadronic tensor components, $W_0^{A\perp\perp}$ and W_0^{A++} etc. modified due to in medium soft scattering?

How they evolve in the final state propagation of the heavy quark?

The final state 3-momentum distribution of the heavy quark in the case without any final state scattering is expected to be a delta function, *i.e.*,

$$\frac{dW_0^{A\mu\nu}}{dl^- d^2 l_\perp} = W_0^{A\mu\nu} \phi_{0\{\mu,\,\nu\}}(l^-, l_\perp)$$
$$= W_0^{A\mu\nu} \delta(l^- - q^-)\, \delta^2(l_\perp)$$

medium modification of Hadronic tensor components



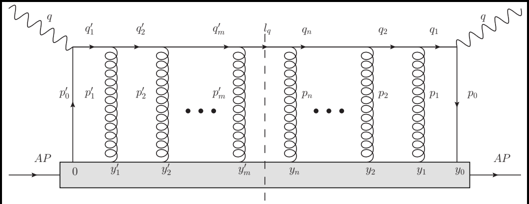
A hard virtual photon strikes a hard heavy quark in the nucleus with momentum p_0' (p_0 in complex conjugate) at location $y_0' = 0$ (y_0 in complex conjugate).

The stuck quark is then sent back through the nucleus and has momentum q_1' (q_1 in the complex conjugate).

The heavy quark scatters off the gluon field within the nuclear medium at locations y_j' with $1 \geq j \geq m$ (y_i in the complex conjugate with $1 \geq i \geq n$).

Through each scattering, the hard parton picks up momenta p_j' (p_i in the complex conjugate). Momentum conservation holds at each vertex.

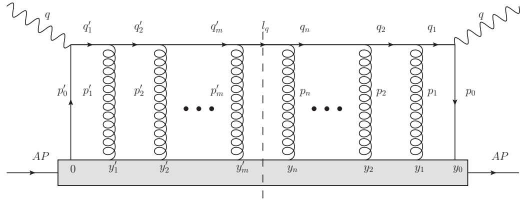
$$q_{i+1} = q_i + p_i = q + \sum_{j=0}^{i} p_j \qquad | \qquad q'_{i+1} = q'_i + p'_i = q + \sum_{j=0}^{i} p'_j$$



$$W^{A\mu\nu}_{mn} \propto \sum_q Q^2_{\mathcal{Q}} \int \frac{d^4 l_q}{(2\pi)^4} (2\pi)\delta^4(l_q^2 - M^2) \int d^4 y_0 e^{iq\cdot y_0}$$

$$\langle A|\bar{\psi}(y_0)\gamma^{\mu} \left(\prod_{i=1}^{n} S_F(q_{j-1} - q_j)\, \Gamma(q_j) \right) \gamma \cdot l_q \left(\prod_{j=m}^{1} \Gamma(q_j)\, S_F(q'_j - q'_{j-1}) \right) \gamma^{\nu} \psi(0)|A\rangle .$$

medium modification of hadronic tensor



$$
\begin{aligned}
W^{A\mu\nu}_{mn} \quad = \quad & \sum_q q_Q^2 \,\mathrm{Tr}\left[\left(\prod_{i=1}^{n} T^{a_i}\right)\left(\prod_{j=m}^{1} T^{a'_j}\right)\right] \int \frac{d^4 l_q}{(2\pi)^4}(2\pi)\delta^{+}(l_q^2 - M^2) \int d^4 y_0 e^{iq\cdot y_0} \\
& \left(\prod_{i=1}^{n}\int d^4 y_i\right)\left(\prod_{j=1}^{m}\int d^4 y'_j\right)\left(\prod_{i=1}^{n}\int \frac{d^4 q_i}{(2\pi)^4} e^{-iq_i\cdot(y_{i-1}-y_i)}\right) e^{-il\cdot(y_n - y'_m)} \left(\prod_{j=1}^{m}\int \frac{d^4 q'_j}{(2\pi)^4} e^{-iq'_j\cdot(y'_j - y'_{j-1})}\right) \\
& \langle A|\bar{\psi}(y_0)\gamma^{\mu}\left(\prod_{i=1}^{n}\frac{\gamma\cdot q_i + M}{q_i^2 - M^2 - i\epsilon}\gamma\cdot A^{a_i}(y_i)\right)\gamma\cdot l_q \left(\prod_{j=m}^{1}\gamma\cdot A^{a'_j}(y'_j)\frac{\gamma\cdot q'_j + M}{q'^2_j - M^2 + i\epsilon}\right)\gamma^{\nu}\psi(0)|A\rangle .
\end{aligned}
$$

mass modification to spin sum

Corrections due to mass occur from two sources :

Numerator : Spin structure of the intermediate propagators.

Denominator : Pole structure of the intermediate propagators.

In $A^- = 0$ gauge :: $A^+ \sim \lambda^2 Q$ and $A_\perp \sim \lambda^3 Q$ $\Rightarrow \gamma \cdot A(y) \approx \gamma^- A^+(y)$.

$$\mathrm{Tr}\left[\gamma^-\gamma^\mu \underbrace{--}_{M} \dots \underbrace{\gamma^- A^+}_{\gamma^\perp A^\perp}(\{q^- + p^-\}\gamma^+ + M)\gamma^- A^+ \dots \gamma^- A^+(\{q^- + p^-\}\gamma^+ + M)\underbrace{\gamma^- A^+}_{\gamma^\perp A^\perp} \dots \underbrace{--}_{M}\gamma^\nu\right] = 0$$

$$\{\gamma^+,\gamma^+\} = \{\gamma^-,\gamma^-\} = 0.$$

As a result, W^{++} survives as leading mass correction. First non-vanishing correction from M dependent terms over $W^{\perp\perp}$ is λ^2 suppressed in $A^- = 0$ gauge.

mass correction to pole structure

Denominator of the i'th propagator

$$
\begin{aligned}
& q_{i+1}^2 - M^2 \\
= \quad & (q + \sum_{j=0}^{i} p_j)^2 - M^2 \\
= \quad & \underbrace{-Q^2 + 2q^+ p_0^- + 2q^- p_0^+}\underbrace{+2q^+ k_i^- + 2q^- k_i^+ + 2p_0^+ k_i^- + 2p_0^- k_i^+ - 2p_0^\perp k_i^\perp - k_i^{\perp\,2}}_{\sim \lambda^2 Q^2}\underbrace{+2k_i^+ k_i^-}_{\sim \lambda^4 Q^2} \\
\sim \quad & 2P^+ q^- \left(\underbrace{-\frac{Q^2 - 2q^+ p_0^-}{2P^+ q^-} + \frac{p_0^+}{P^+}} + \frac{(-Q^2 + x_0 P^+ q^-)}{2(q^-)^2} \underbrace{\frac{k_i^-}{P^+}}_{\lambda^2 Q^2} + \frac{(q^- + p^-)}{q^-} \underbrace{\frac{k_i^+}{P^+}}_{\lambda^2 Q^2} - \frac{P^+}{2q^-} \underbrace{\frac{k_i^{\perp\,2}}{P^{+\,2}}}_{\lambda^2 Q^2} \right) \\
\equiv \quad & 2P^+ q^- \left(-\tau + x_{+0} + a_- \bar{x}_{-i} + a_+ \bar{x}_{+i} - a_\perp \bar{x}_{\perp i} \right)
\end{aligned}
$$

$$k_i^\perp \sim \lambda Q, \quad k_i^+, k_i^- \sim \lambda^2 Q$$

hadronic tensor

$$
\begin{aligned}
W^{A,\mu\nu}_{M,mn} \quad = \quad & \sum_q \mathcal{Q}_q^2 \, g^{n+m} \, \frac{1}{N_c} \mathrm{Tr} \left[\left(\prod_{i=1}^{n} T^{a_i} \right) \left(\prod_{i=1}^{m} T^{a'_j} \right) \right] \\
& \int \frac{dl_q^-}{2\pi} \, 2\pi \, \delta \left(l_q^- - q^- - p_0^- - k_n^- \right) \int \frac{dl_q^\perp}{2\pi} \, 2\pi \, \delta \left(l_q^\perp - p_0^\perp - k_n^\perp \right) \blacktriangleleft \; q^\perp = 0 \\
& \prod_{i=0}^{n} \int dy_i^+ dy_i^- d^2 y_i^\perp \; \prod_{j=1}^{m} \int dy'^+_i dy'^-_i d^2 y'^\perp_i \; \blacktriangleleft \; y'_0 \equiv 0 \\
& \prod_{i=0}^{n} \int \frac{dp_i^-}{2\pi} \frac{d^2 p_i^\perp}{(2\pi)^2} \; \prod_{j=0}^{m-1} \int \frac{dp'^-_i}{2\pi} \frac{d^2 p'^\perp_i}{(2\pi)^2} \; \blacktriangleleft \; p'_m \equiv p_n \\
& \prod_{i=0}^{n} e^{-ip_i^- y_i^+} e^{-ip_i^\perp y_i^\perp} \prod_{j=1}^{m} e^{+ip'^-_j y'^+_j} e^{+ip'^\perp_j y'^\perp_j} \; \blacktriangleleft \; y'_0 \equiv 0 \\
& e^{+ix_B P^+ y_0^-} \left(\prod_{i=1}^{n} e^{i(\sigma_- x_{-,i} - \sigma_\perp x_{\perp,i}) P^+ y_i^-} \right) i^n \prod_{i=1}^{n} \theta(y_i^- - y_{i-1}^-) \\
& \left(\prod_{i=1}^{m} e^{-i(\sigma_- x'_{-,i} - \sigma_\perp x'_{\perp,i}) P^+ y'^-_i} \right) (-i)^m \prod_{i=1}^{m} \theta(y'^-_j - y'^-_{j-1}) \; \blacktriangleleft \; y'_0 \equiv 0 \\
& \left(\frac{Q^2}{2P^+ q^-} \right)^{-2} \left(1 + \frac{4M^2}{Q^2} \right)^{-1} \left(-g^{\mu\perp} g^{\nu\perp} + \frac{M^2}{(q^- + p_0^-)(q'^- + p'^-_0)} g^{\mu -} g^{\nu -} \right) \\
& \langle A | \, \overline{\psi}(y_0) \frac{\gamma^+}{2} \left(\prod_{i=1}^{n} A^{+a_i}(y_i) \right) \left(\prod_{i=1}^{m} A^{+a'_j}(y'_i) \right) \psi(0) | A \rangle
\end{aligned}
$$

approximations and other issues

- Its evident that when $M \sim Q$, the hadronic tensor elements $W^{A\perp\perp}_{mn}$ and W^{A++}_{mn} are of same order (W^{A++}_{mn} was $\mathcal{O}(\lambda^2)$ suppressed for light quark, $M \sim \lambda^2 Q$)
- W^{A--}_{mn}, W^{A+-}_{mn} did not survive at this order. This might be an artifact coming from the high energy approximation taken. One however could easily recover them, thanks to current conservation relations.

$$
\begin{aligned}
q^- W_0^{++} + q^+ W_0^{-+} &= 0 \\
q^- W_0^{+-} + q^+ W_0^{--} &= 0 \\
q^- W_0^{++} + q^+ W_0^{+-} &= 0 \\
q^- W_0^{-+} + q^+ W_0^{--} &= 0
\end{aligned}
$$

$$
W^{++} = -\frac{q^+}{q^-} W^{-+} = -\frac{q^-}{q^+} W^{+-} = W^{--}
$$

length enhancement

$$Y_i = (y_i + y_i')/2, \quad \delta y_i = y_i - y_i'.$$

Since both δy_i^- and δy_{i-1}^- are within the nucleon size (small compared to the nucleus size Y^-), we may simplify it as,

$$\theta(y_i^- - y_{i-1}^-)\theta({y'}_i^- - {y'}_{i-1}^-) = \theta(Y_i^- - Y_{i-1}^-).$$

The time-ordered product of θ-functions gives,

$$\begin{aligned} & \left(\prod_{i=1}^{n} \int_0^{L^-} dY_i^- \theta(Y_i^- - Y_{i-1}^-) \right) \\ = \quad & \frac{1}{n!} \left(\prod_{i=1}^{n} \int_0^{L^-} dY_i^- \right), \end{aligned}$$

where L^- is the extent of the nucleus size.

Medium modified hadronic tensor differential contain $\perp$ contribution and $-$ contribution in equal footing.

$$
\frac{dW^{A\mu\nu}_{nn}}{d^3 l_q} = \sum_q \; Q_q^2 \left(-g_\perp^{\mu\nu} + \frac{M^2}{(q^- + p^-)^2}\, g^{\mu -} g^{\nu -} \right) \; A\, C_p^A \int dy_0^- \, e^{-ix_B p^+ y_0^-} \langle p | \bar{\psi}(y_0) \frac{\gamma^+}{2} \psi(0) | p \rangle
$$

$$
\frac{1}{n!} \prod_{i=1}^{n} \left(\int_0^{L^-} dY_i^- \int d\delta y_i^- \int d^3 \delta y_i \int \frac{d^3 p_i}{(2\pi)^3} \frac{\rho}{2p^+} g^2 \frac{C_F}{N_c^2 - 1} e^{-i\vec{p}_i \cdot \delta \vec{y}_i} \langle p | A^+(\delta y_i) A^+(0) | p \rangle \right)
$$

$$
\left(\prod_{i=1}^{n} e^{i(\sigma_{-} x_{-,i} - \sigma_{\perp} x_{\perp,i}) P^+ y_i^-} \right) \delta^3 \left(\vec{l}_q - \vec{q} - \sum_{i=1}^{n} \vec{p}_i \right)
$$

resummation over multiple scatterings

Taylor expands the hard part in terms of the exchanged momenta $\vec{p}_i$ around $\vec{p}_i = 0$,

$$H(q^-, p^+, p^-, p_i^\alpha) = \prod_{i=1}^{n} \left[(H)_{\vec{p}_1 \cdots \vec{p}_n = 0} + p_i^\alpha \left(\frac{\partial}{\partial p_i^\alpha} H \right)_{\vec{p}_1 \cdots \vec{p}_n = 0} + \frac{1}{2} p_i^\alpha p_i^\beta \left(\frac{\partial}{\partial p_i^\alpha} \frac{\partial}{\partial p_i^\beta} H \right)_{\vec{p}_1 \cdots \vec{p}_n = 0} + \cdots \right].$$

In the above equation, α, β represents both "$-$" and "$\perp$".

resummation over multiple scatterings

The factors of exchanged momentum may be converted into the appropriate derivatives over relative position,

$$e^{-i\vec{p}_i\cdot\delta\vec{y}_i}\langle p|A^{+}(\delta\vec{y}_i)A^{+}(0)|p\rangle p_i^{\alpha}\frac{\partial}{\partial p_i^{\alpha}} = e^{-i\vec{p}_i\cdot\delta\vec{y}_i}(-i)\langle p|\partial^{\alpha}A^{+}(\delta\vec{y}_i)A^{+}(0)|p\rangle\frac{\partial}{\partial p_i^{\alpha}},$$

$$e^{-i\vec{p}_i\cdot\delta\vec{y}_i}\langle p|A^{+}(\delta\vec{y}_i)A^{+}(0)|p\rangle p_i^{\alpha}p_i^{\beta}\frac{\partial}{\partial p_i^{\alpha}}\frac{\partial}{\partial p_i^{\beta}} = e^{-i\vec{p}_i\cdot\delta\vec{y}_i}\langle p|\partial^{\alpha}A^{+}(\delta\vec{y}_i)\partial^{\beta}A^{+}(0)|p\rangle\frac{\partial}{\partial p_i^{\alpha}}\frac{\partial}{\partial p_i^{\beta}}.$$

resummation over multiple scatterings

$$\frac{dW^{A\perp\perp}_{nn}}{d^3 l_q} = W_0^{A\perp\perp} \times \frac{1}{n!}\left(\prod_{i=1}^{n}\int_0^{L^-} dY_i^- \left[-\mathcal{D}^{\perp\perp}_{L1}\frac{\partial}{\partial p_i^-} + \frac{1}{2}\mathcal{D}^{\perp\perp}_{L2}\frac{\partial^2}{\partial^2 p_i^-} + \frac{1}{2}\mathcal{D}^{\perp\perp}_{D2}\nabla^2_{p_{i\perp}}\right]\right)\delta^3\left(\vec{l}_q - \vec{q} - \sum_{i=1}^{n}\vec{p}_i\right)$$

The jet transport coefficients $\mathcal{D}^{\perp\perp}_{L1}$, $\mathcal{D}^{\perp\perp}_{L2}$ and $\mathcal{D}^{\perp\perp}_{T2}$ are defined as,

$$\mathcal{D}_{L1} = g^2\frac{C_F}{N_c^2-1}\int dy^- \frac{\rho}{2p^+}\langle p|i\partial^- A^+(y^-)A^+(0)|p\rangle,$$

$$\mathcal{D}_{L2} = g^2\frac{C_F}{N_c^2-1}\int dy^- \frac{\rho}{2p^+}\langle p|\partial^- A^+(y^-)\partial^- A^+(0)|p\rangle,$$

$$\mathcal{D}_{T2} = g^2\frac{C_F}{N_c^2-1}\int dy^- \frac{\rho}{2p^+}\langle p|\partial^\perp A^+(y^-)\partial^\perp A^+(0)|p\rangle.$$

These coefficients $\mathcal{D}^{\perp\perp}_{L1}$, $\mathcal{D}^{\perp\perp}_{L2}$ and $\mathcal{D}^{\perp\perp}_{D2}$ are, up to an overall factor, related to elastic energy loss rate $\hat{e}$, the diffusion in longitudinal momenta $\hat{e}_2$ and the diffusion in transverse momenta $\hat{q}$.

resummation over an arbitrary number of multiple scatterings

$$\frac{dW^{A\perp\perp}}{d^3 l_q} = \sum_{n=0}^{\infty} \frac{dW^{A\perp\perp}_{nn}}{d^3 l_q} = W_0^{A\perp\perp} \phi_{\{\perp\perp\}}(L^-, l_q^-, \vec{l}_{q\perp}),$$

where we have defined the final quark distribution function $\phi_{\{\perp\perp\}}(L^-, l_q^-, \vec{l}_{q\perp})$,

$$\phi_{\{\perp\perp\}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=1}^{n} \int_0^{L^-} dY_i^- \left[-\mathcal{D}_{L1} \frac{\partial}{\partial p_i^-} + \frac{1}{2} \mathcal{D}_{L2} \frac{\partial^2}{\partial^2 p_i^-} + \frac{1}{2} \mathcal{D}_{T2} \nabla^2_{p_{i\perp}} \right] \right) \delta(l_q^- - q^- - p_0^-)\, \delta^2(\vec{l}_{q\perp})$$

The quark distribution function, after resumming over an arbitrary number of scatterings,

$$\phi_{\{\perp,\perp\}}(L^-, l_q^-, \vec{l}_{q\perp}) = \exp\left(L^- \left[\mathcal{D}_{L1} \frac{\partial}{\partial l_q^-} + \frac{1}{2} \mathcal{D}_{L2} \frac{\partial^2}{\partial^2 l_q^-} + \frac{1}{2} \mathcal{D}_{T2} \nabla^2_{l_{q\perp}} \right] \right) \delta(l_q^- - q^- - p_0^-)\, \delta^2(\vec{l}_{q\perp})$$

Connection to transport coefficients

The distribution function $\phi(L^-, l_q^-, \vec{l}_{q\perp})$ for the final quark satisfies the following evolution equation,

$$\frac{\partial \phi}{\partial L^-} = \left[\mathcal{D}_{L1} \frac{\partial}{\partial l_q^-} + \frac{1}{2} \mathcal{D}_{L2} \frac{\partial^2}{\partial^2 l_q^-} + \frac{1}{2} \mathcal{D}_{T2} \nabla^2_{l_{q\perp}} \right] \phi(L^-, l_q^-, \vec{l}_{q\perp}).$$

The three terms in the above evolution equation represent the contributions from longitudinal momentum loss and diffusion, and the diffusion of transverse momentum.

$$\phi_{\{\perp,\perp\}}(L^-, l_q^-, \vec{l}_{q\perp}) = \frac{1}{\sqrt{2\pi \mathcal{D}_{L2} L^-}} \exp\left[-\frac{(l_q^- - q^- + \mathcal{D}_{L1} L^-)^2}{2\mathcal{D}_{L2} L^-} \right] \frac{1}{2\pi \mathcal{D}_{T2} L^-} \exp\left[\frac{-l_{q\perp}^2}{2\mathcal{D}_{T2} L^-} \right].$$

From the above solution, one may obtain

$$\langle l_q^- \rangle = q^- - \mathcal{D}_{L1} L^-, \quad \langle (l_q^-)^2 \rangle - \langle l_q^- \rangle^2 = \mathcal{D}_{L2} L^-, \quad \langle l_{q\perp}^2 \rangle = 2\mathcal{D}_{T2} L^-.$$

The coefficients D_{L1}, D_{L2} and D_{T2} are defined on the light cone, and they are related to elastic energy loss rate $\hat{e} = dE/dt$, the diffusion of elastic energy loss $\hat{e}_2 = d(\Delta E)^2/dt$, and the transverse momentum diffusion rate $\hat{q} = d(\Delta p_T)^2/dt$ as

$$\hat{e} = \mathcal{D}_{L1}, \quad \hat{e}_2 = \mathcal{D}_{L2}/\sqrt{2}, \quad \hat{q} = 2\sqrt{2}\mathcal{D}_{T2}$$

Part II :: Wide angle soft radiation from heavy flavor at Gribov limit

Wide angle soft radiation

Expression for wide angle soft radiation spectrum from heavy flavor at Gribov limit without the eikonal approximation

$$\frac{1}{\sigma}\frac{dN_g}{dxdk_\perp^2} \quad \propto \quad \frac{1}{x}\frac{1}{k_\perp^2}\underbrace{\left[\sum_{n=2,1,0} \mathcal{C}_n \; e^{2(n-1)\eta_g}\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^n \quad\right]}_{\mathcal{W}(x,k_\perp^2)}$$

Coefficient $\mathcal{C}_n$ contain the non-eikonal parameter $\frac{q_\perp}{E}$. In the limit $\frac{q_\perp}{E} \to 0$ it reproduce well know eikonal spectrum.

arXive: arXiv:1307.6931 with Trambak Bhattacharyya, Surasree Mazumder.

wide angle soft radiation

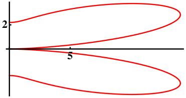


Figure: Polar plot of $\mathcal{W}(\theta_g, \omega)$ (at a typical ω) with $\zeta(= q_\perp/\sqrt{s}) = 0$ for a 10 GeV Charm jet.

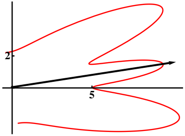


Figure: Polar plot of $\mathcal{W}(\theta_g, \omega)$ (at a typical ω) with $\zeta(= q_\perp/\sqrt{s}) = 0.15$ for a 10 GeV Charm jet.

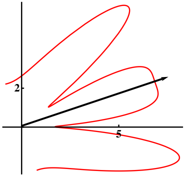


Figure: Polar plot of $\mathcal{W}(\theta_g, \omega)$ (at a typical ω) with $\zeta(= q_\perp/\sqrt{s}) = 0.30$ for a 10 GeV Charm jet.

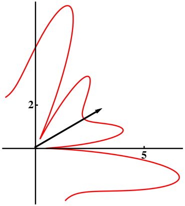
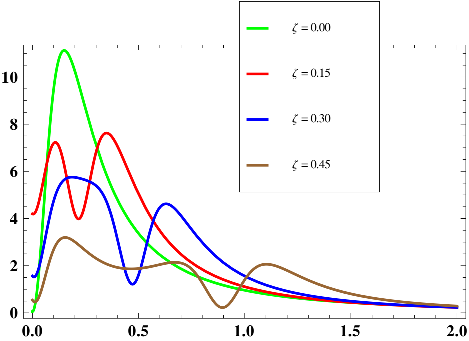


Figure: Polar plot of $\mathcal{W}(\theta_g, \omega)$ (at a typical ω) with $\zeta(= q_\perp/\sqrt{s}) = 0.45$ for a 10 GeV Charm jet.

frame



Transverse momentum (of emitted gluons) ($k_\perp$) dependence of the factor $\mathcal{W}[x, k_\perp]$ for different $\zeta(= q_\perp/\sqrt{s})$.