Thermalization in heavy ion collisions from holography

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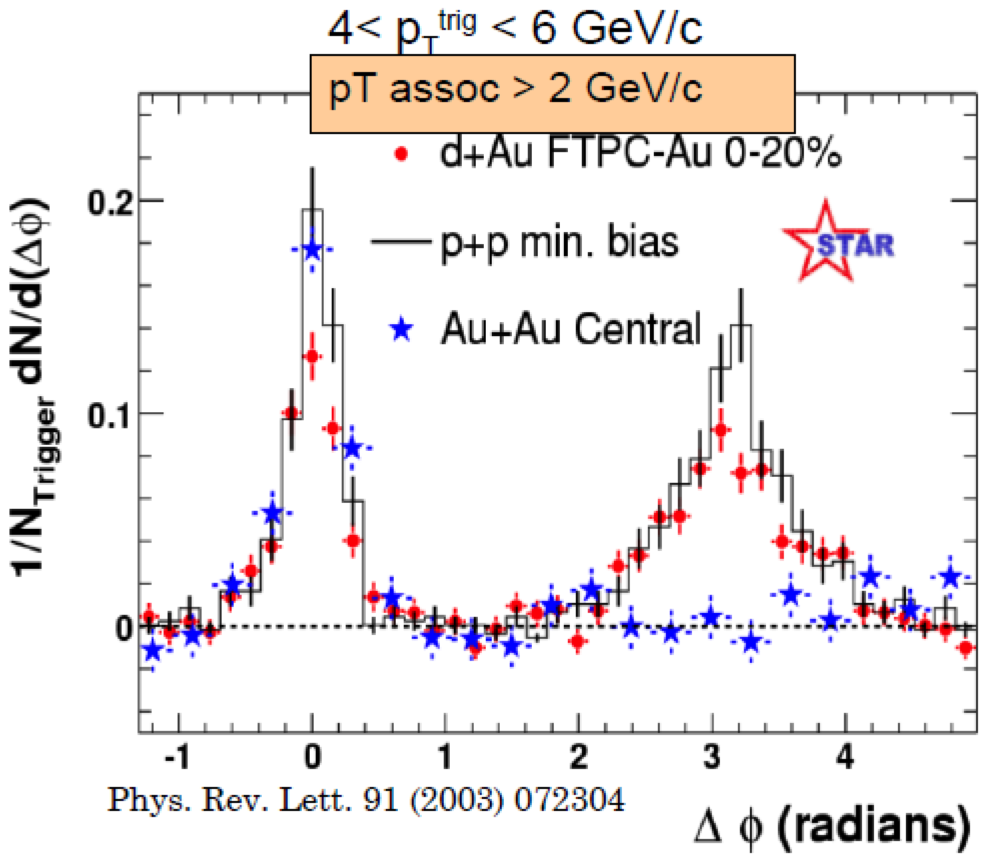
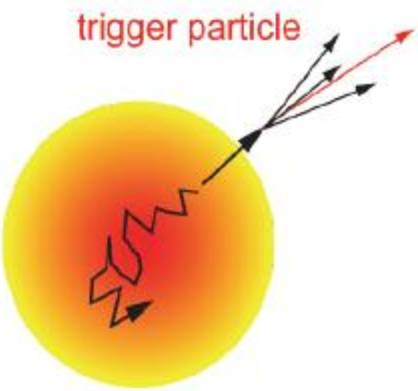
Outline

- Introduction to thermalization problem.
- The theoretical tool: AdS/CFT.
- A gravity dual of heavy ion collisions and hologram on the boundary.
- Gravitational collapse model and thermalization
- Two point function as probe of thermalization: top-down thermalization scenario?
- Quasi-static approximation and geometric optics approximation
- Summary

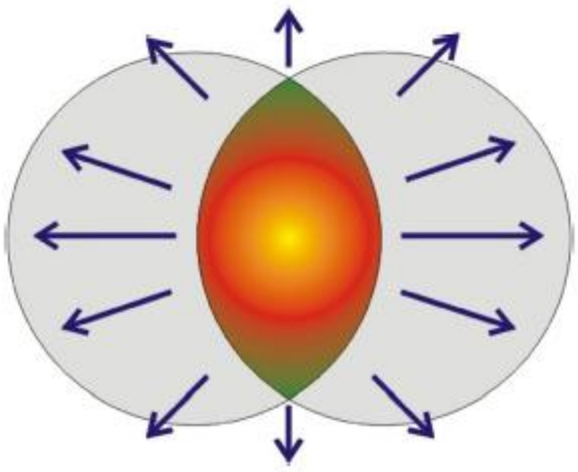
Evidence for Quark Gluon Plasma (QGP)



Jet quenching



Evidence for QGP



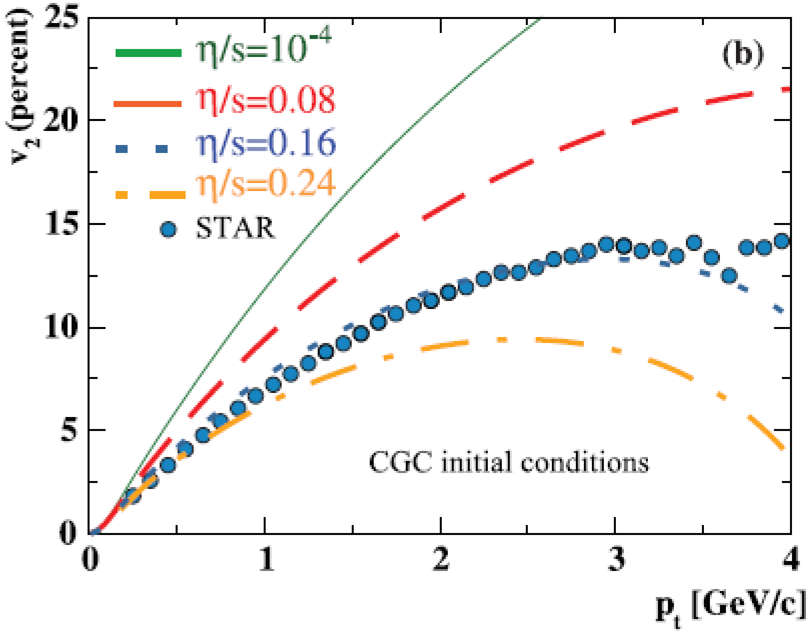
$$E\frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_{RP})]\right)$$

φ the azimuthal angle,

Ψ_{RP} the reaction plane angle

Initial spatial anisotropy ➔ azimuthal anisotropy in particle production

Hydrodynamic description of Elliptic flow v2



$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

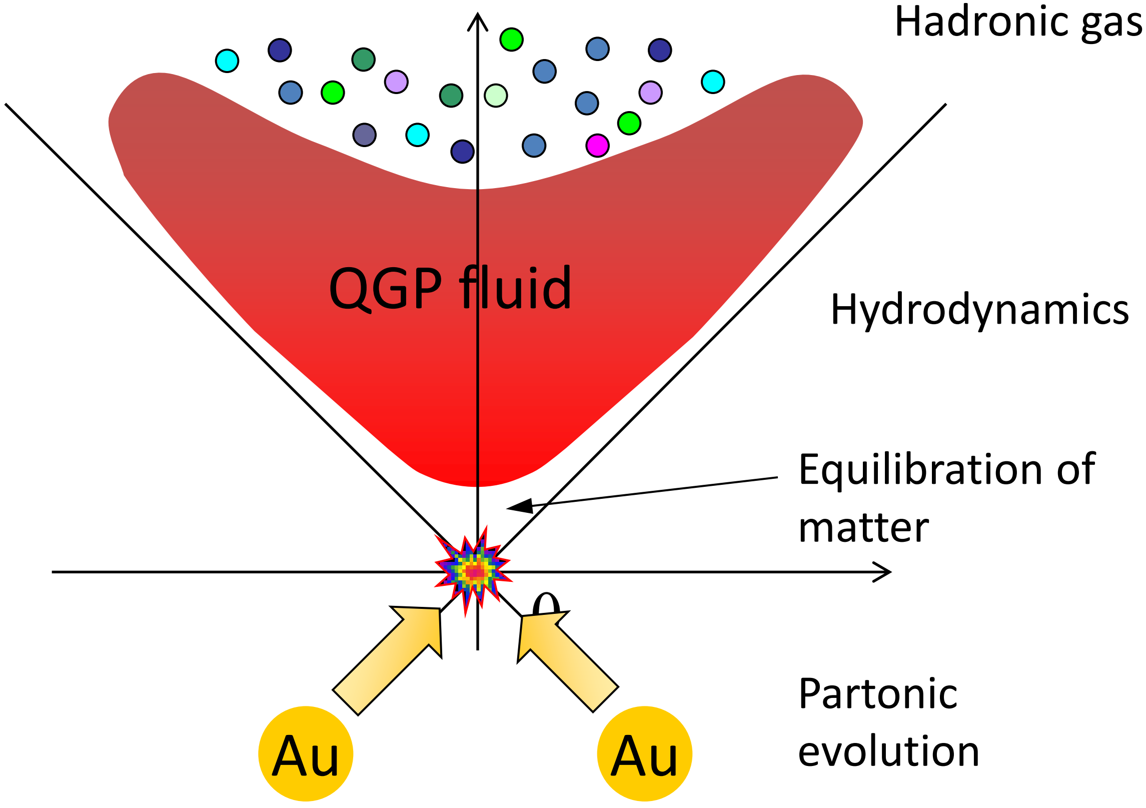
$$\tau^{\mu\nu} = -\eta P^{\mu\alpha} P^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - (\zeta - \tfrac{2}{3}\eta) P^{\mu\nu} \partial \cdot u,$$

η: shear viscosity
ζ: bulk viscosity

Hydrodynamics requires field reaches local thermal equilirium

One input parameter of hydrodynamics:
Onset time of hydrodynamics:
Thermalization time ~ 0.5 fm/c

Stages of heavy ion collisions



thermalization time scale 0.5fm/c

$\Rightarrow$

Fast thermalization, suggests strong interaction (QCD at strong coupling regime)

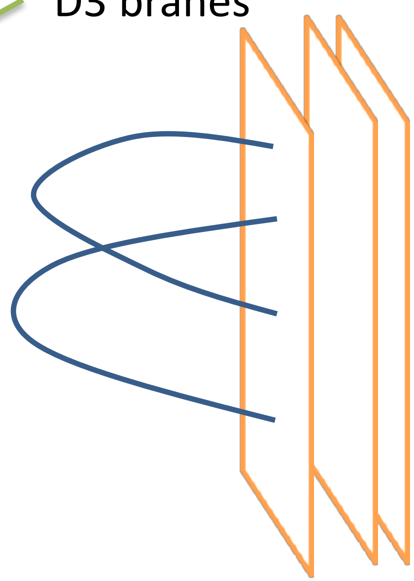
AdS/CFT correspondence

Large N_c , strong coupling λ limit of $\mathcal{N}$=4 SYM

$\Longleftrightarrow$

Classical SUGRA in AdS background

low energy theory of N_c D3 branes



3+1D worldvolume fields: SU(N) gluons etc

D3 brane can be viewed as soliton in 10D, alternatively described by SUGRA fields: graviton, dilaton etc

AdS/CFT preliminaries

Pure AdS $$ds^2 = \frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2)$$

N=4 SYM at zero temperature (vacuum)

AdS-Schwarzschild $$ds^2 = \frac{L^2}{z^2}(-f(z)dt^2 + d\vec{x}^2 + dz^2 / f(z))$$

$$f = 1 - \frac{z^4}{z_h^4}$$

N=4 SYM at temperature (plasma) $$T = \frac{1}{\pi z_h}$$

bulk field ⟺ boundary operator

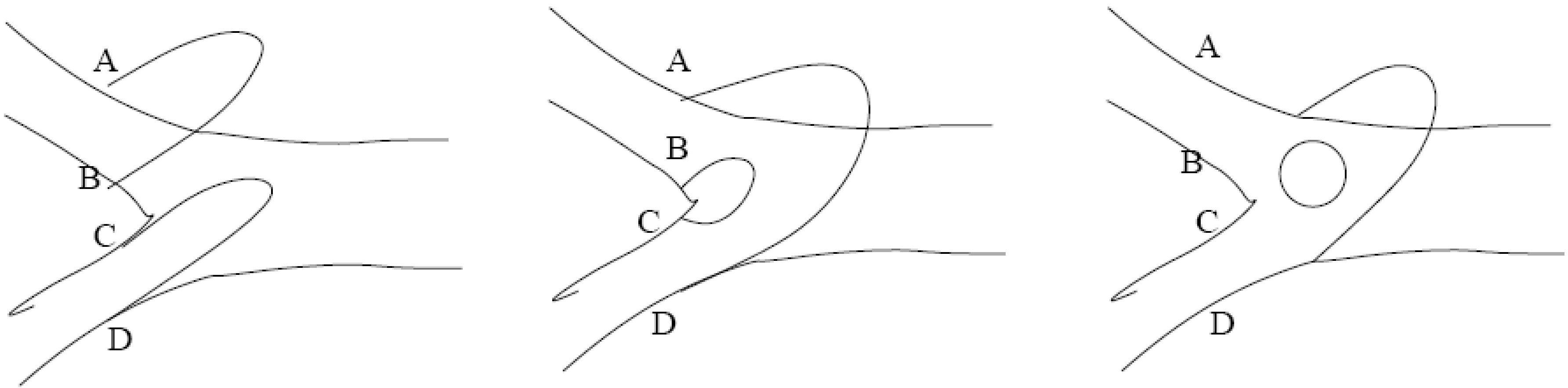
dictionary

ϕ — $O = TrF^2 + \ldots$

A_μ — J^μ

$h_{\mu\nu}$ — $T^{\mu\nu}$

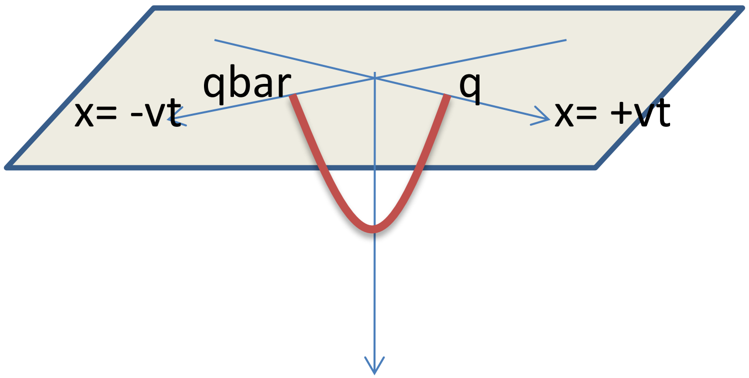
A gravity dual of heavy ion collisions



Example: a collision of two open string (mesons) results in one closed string (debris) and one open string (separating quark/anti-quark)

Shuryak, Sin, Zahed 05

Scaling string



separating quark/anti-quark

Scaling solution exists for v < 0.462

$$S = -\frac{R^2}{2\pi\alpha'}\int \frac{\tau\, d\tau\, dy}{z^2}\sqrt{1-\left(\frac{\partial z}{\partial\tau}\right)^2+\frac{\left(\frac{\partial z}{\partial y}\right)^2}{\tau^2}}$$

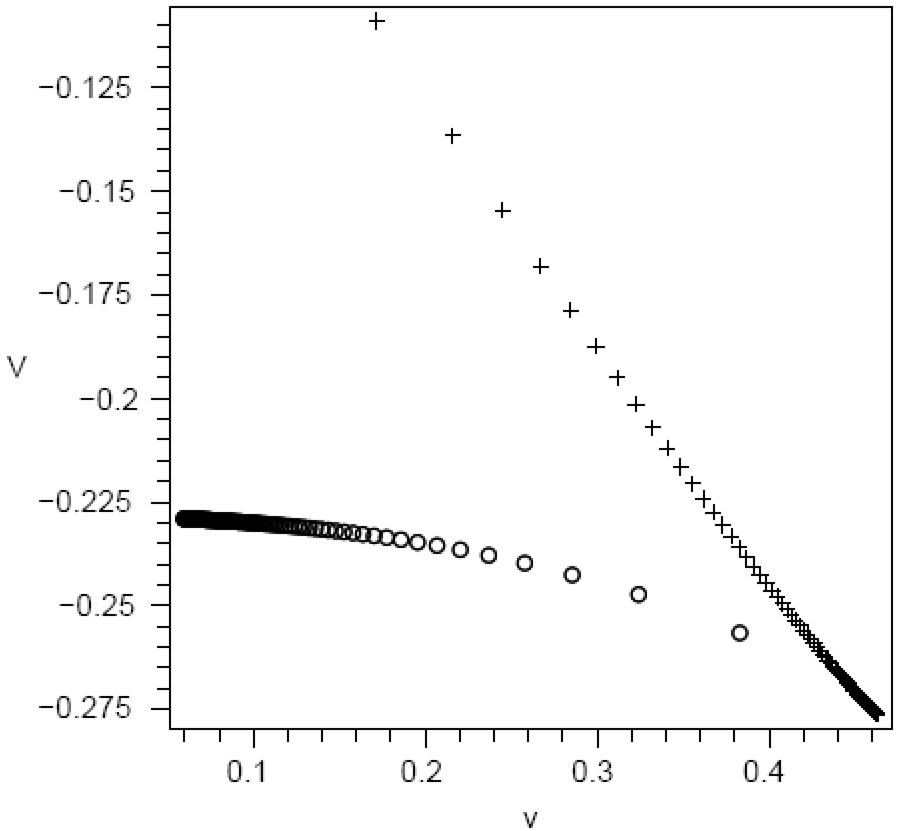
$$z(\tau, y) = \frac{\tau}{f(y)}$$

$$y = f_0\sqrt{\frac{f_0^2-1}{2f_0^2-1}}F\left(\sqrt{\frac{f^2-f_0^2}{f^2-1}},\frac{f_0}{\sqrt{2f_0^2-1}}\right)$$

$$-\frac{1}{f_0}\sqrt{\frac{(f_0^2-1)^3}{(2f_0^2-1)}}\Pi\left(\sqrt{\frac{f^2-f_0^2}{f^2-1}},\frac{1}{f_0^2},\frac{f_0}{\sqrt{2f_0^2-1}}\right)$$

SL, Shuryak PRD 07

v-dependent potential



Ampere's law at strong coupling

$$V = 0.2285\,\frac{(1 + 0.6830\,v^2)\,\sqrt{g^2\,N}}{L}$$

Hologram of scaling string

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu}$$

Stress energy tensor on the boundary CFT: $T_{mn} = 2/\kappa^2 * Q_{mn}$

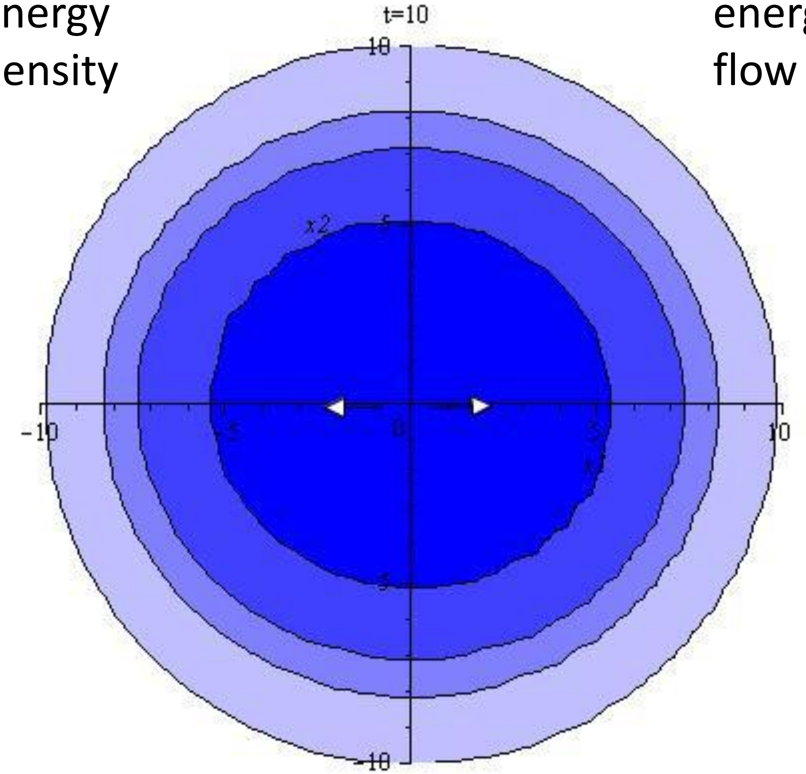
$$Q_{mn}(t', x') = \int P_R(t' - t, x' - x, z) s_{mn}(z, t, x) dz dt d^3x$$

$$P_R = \frac{12iz}{(2\pi)^2} \left[\frac{1}{(t^2 - r^2 - z^2 + i\epsilon)^4} - \frac{1}{(t^2 - r^2 - z^2 - i\epsilon)^4} \right] \theta(t - r)$$

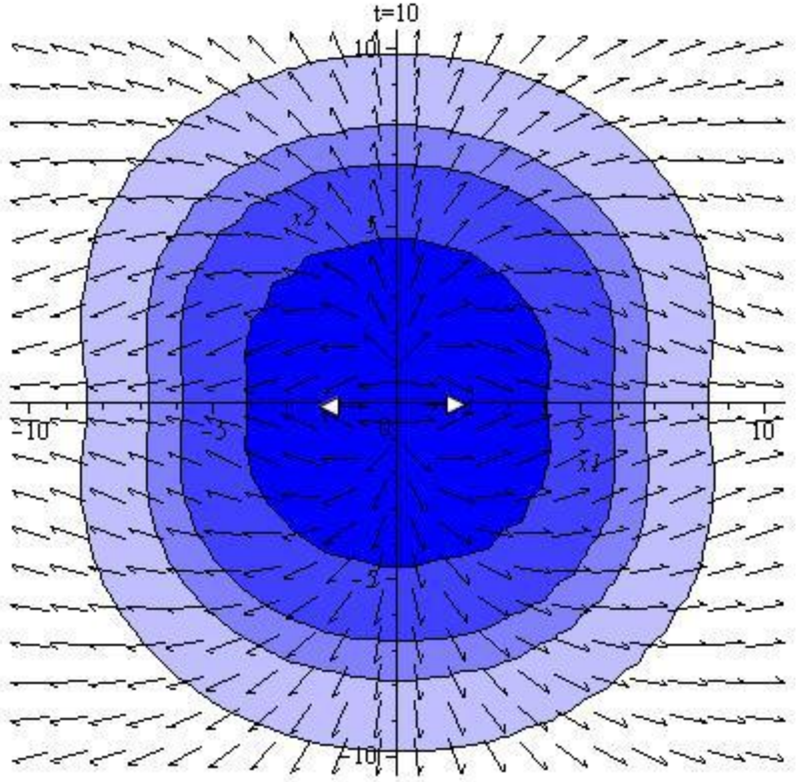
Propogator: P_R
Source: s_{mn} linear combination of $T_{\mu\nu}$

Hologram of scaling string

energy
density



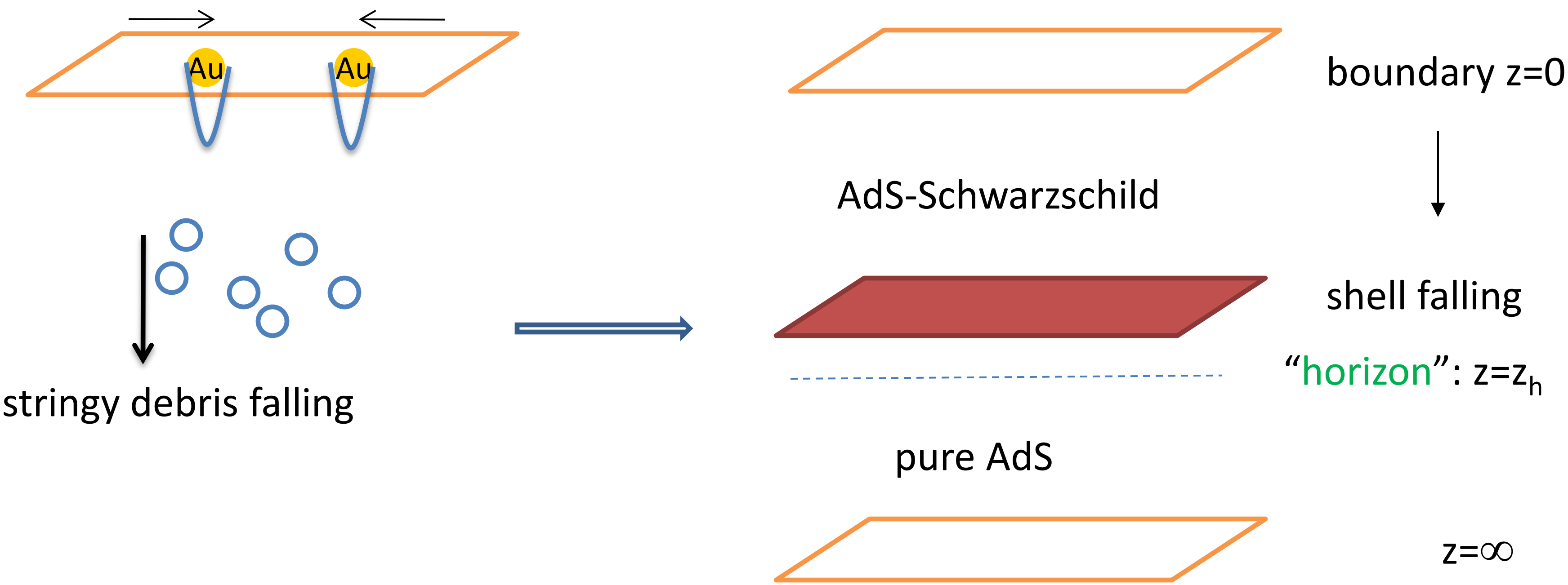
energy
flow



No jet like structure! Nearly isotropic distribution. quark anti-quark pair remain strongly correlated all the time.

SL, Shuryak PRD 07

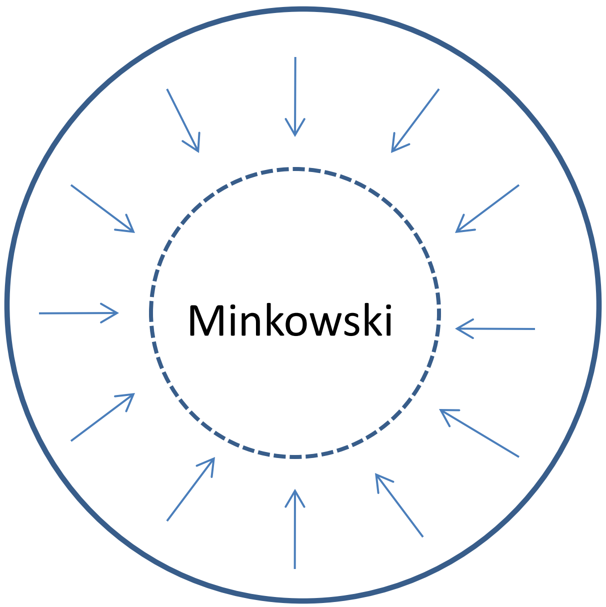
Gravitational collapse dual to thermalization

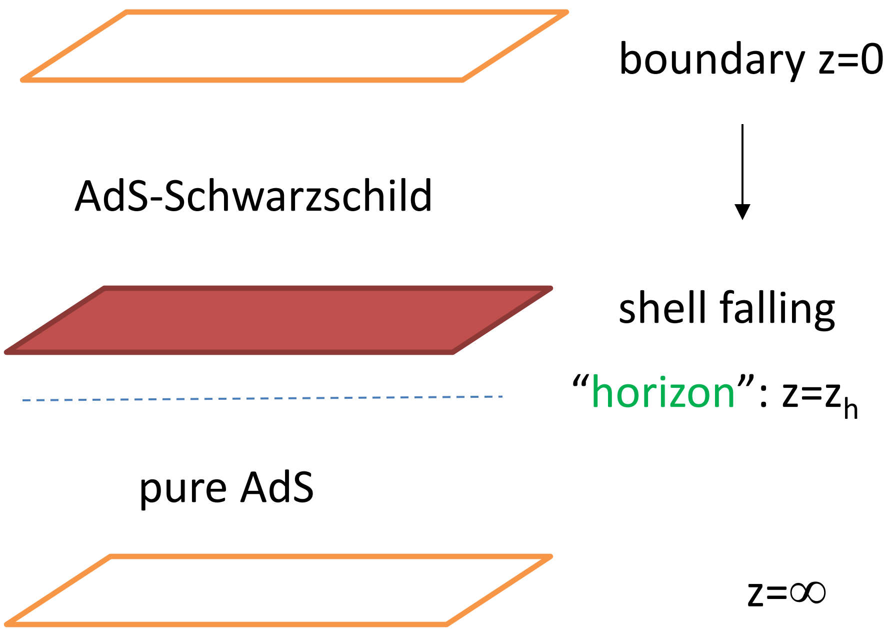


SL, Shuryak PRD 08

Gravitational Collapse in AdS & flat space

Schwarzschild





Trajectory of falling shell from Israel junction condition

Israel junction condition

$$[K_{ij} - \gamma_{ij} K] = \kappa S_{ij} \,, \qquad \{K_{ij}\} S^{ij} = 0.$$

K_{ij}: extrinsic curvature
γ_{ij}: intrinsic curvature

$$S_{ij} = (\epsilon(z) + p(z)) u_i u_j + p(z) \gamma_{ij}.$$

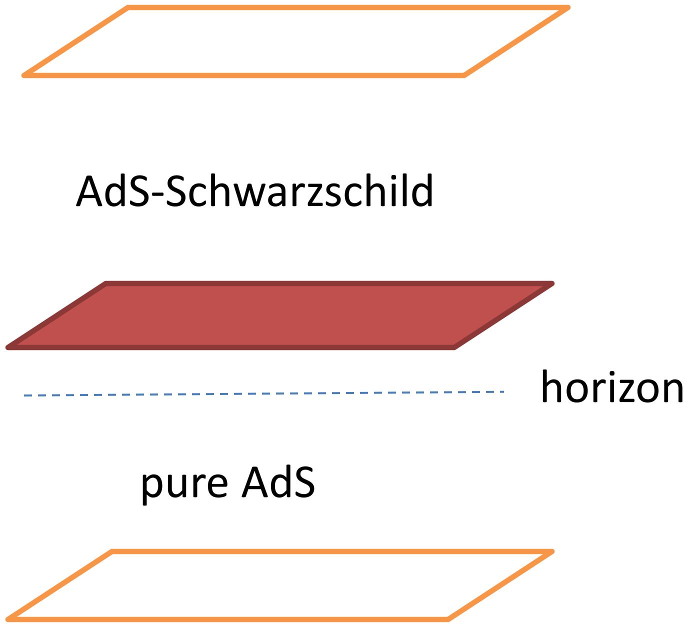
Equation of State $\epsilon = \frac{p}{a}$

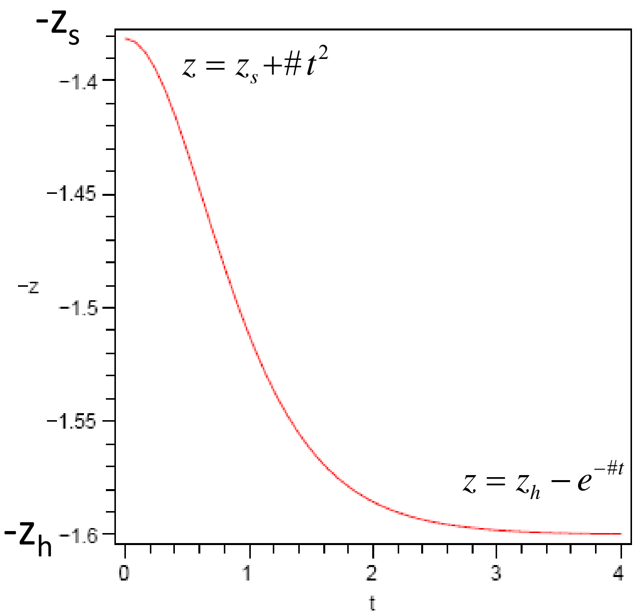
$$\Rightarrow \quad \dot{z} = \sqrt{\frac{1}{4}\left(bz^4 + \frac{1}{bz_h^4}\right)^2 - 1} \,, \quad \dot{t}_f = \frac{\sqrt{f + \dot{z}^2}}{f} = \frac{\frac{1}{bz_h^4} - bz^4}{2f}.$$

$$bz_s^4 + \frac{1}{bz_h^4} = 2.$$

z_s: initial shell position (intrinsic scale)
b: “energy density”
z_h: horizon position (temperature)

Trajectory of falling shell from Israel junction condition





One-point function vs Two-point function

$$T_{\mu\nu} = diag(\varepsilon, p, p, p) = T_{\mu\nu}^{thermal}$$

One-point function same as thermal counterpart: thermalizes instantaneously

Focus on →

$$\langle T^{\mu\nu}(x) T^{\lambda\rho}(y) \rangle \neq \langle T^{\mu\nu}(x) T^{\lambda\rho}(y) \rangle^{thermal}$$

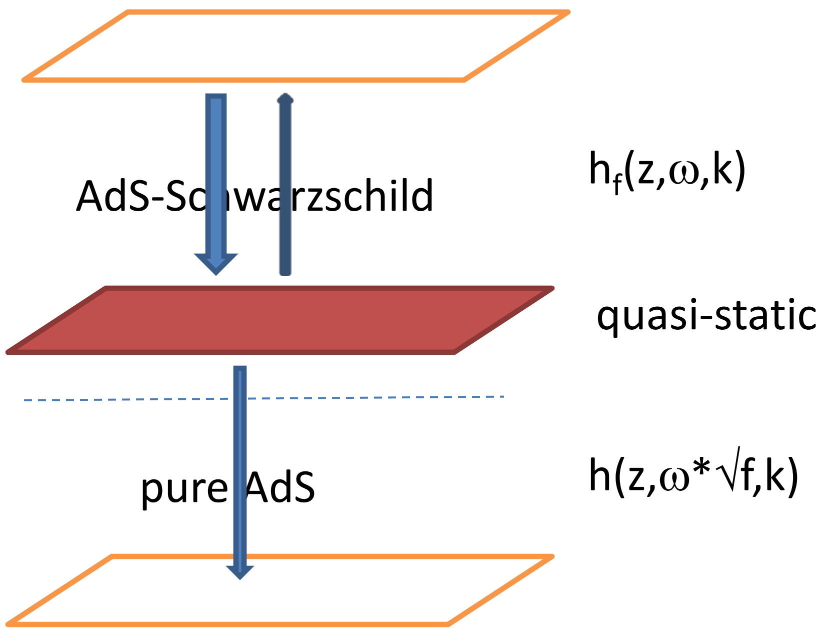
$$\langle O(x) O(y) \rangle \neq \langle O(x) O(y) \rangle^{thermal}$$

$$\cdots$$

Two-point function in general differs from thermal counterpart, measures approach to equilibrium

Similar model (Chesler, Teaney 11) also indicates two-point function takes longer time to thermalizes than one-point function

Correlator for stress tensor from bulk graviton in quasi-static approximation

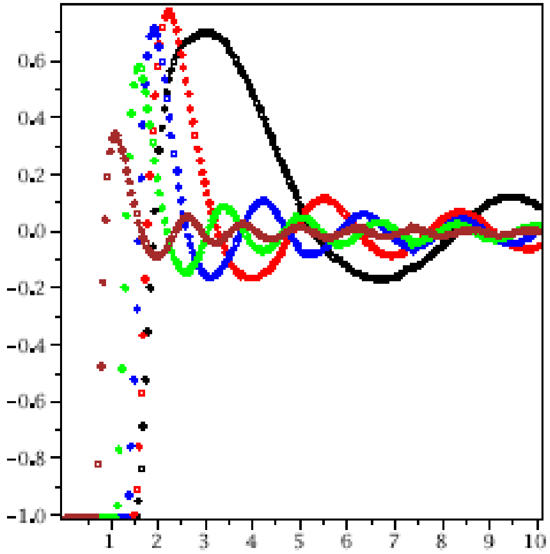


$$G^R(t,t',x) = \theta(t-t')\langle[O(t,x),O(t',0)]\rangle$$
$$= G^R(t-t',x) = \int d\omega d^3k e^{-i\omega t+ikx} G^R(\omega,k)$$

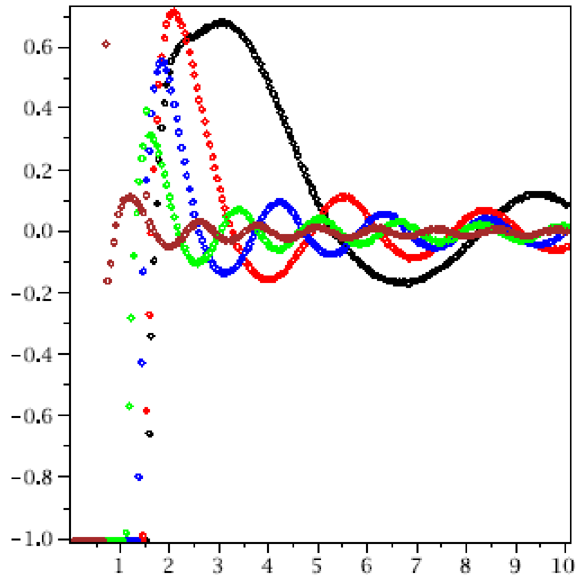
$$G^R(\omega,k) \to G^R_{thermal}(\omega,k)$$

Quasi-static approximation justified when ω not too small, state not too close to equilibrium

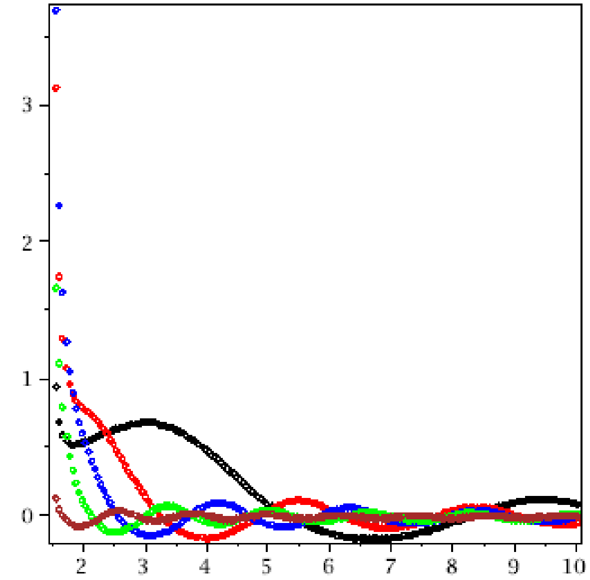
Deviation of spectral function from thermal counterpart



R_{xy}-ω, stress



R_{tx}-ω, momentum density



R_{tt}-ω, energy density

$$\chi = -2\,\mathrm{Im}\,G^R(\omega)$$

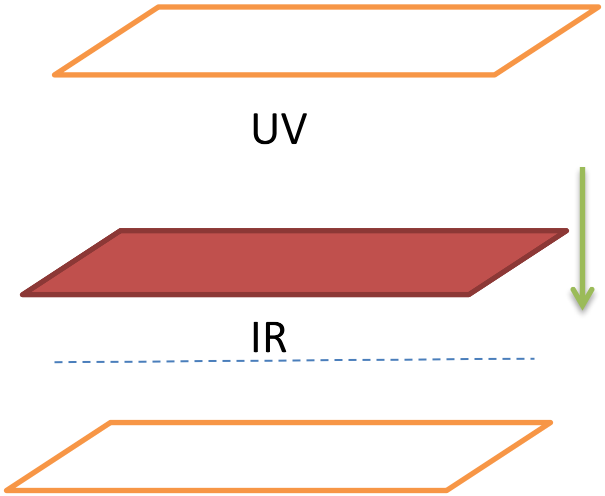
$$R = \frac{\chi - \chi_{thermal}}{\chi_{thermal}}$$

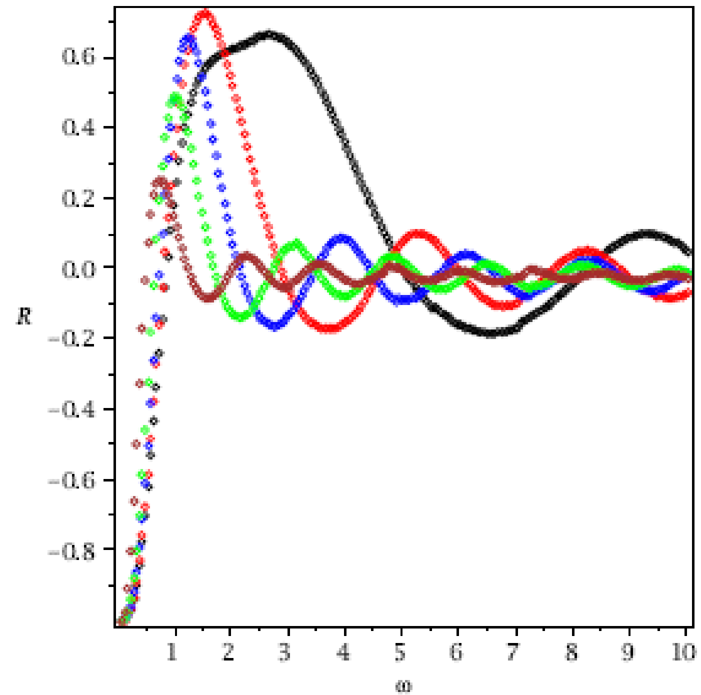
black red blue green brown

Far from equlibrium → Near equlibrium

Deviation: general oscillations, increasing number of oscillations and shrinking amplitude.

Top-down thermalization scenario





In contrast to weak coupling:
bottom-up scenario
(Baier, Mueller, Schiff, Son 11)

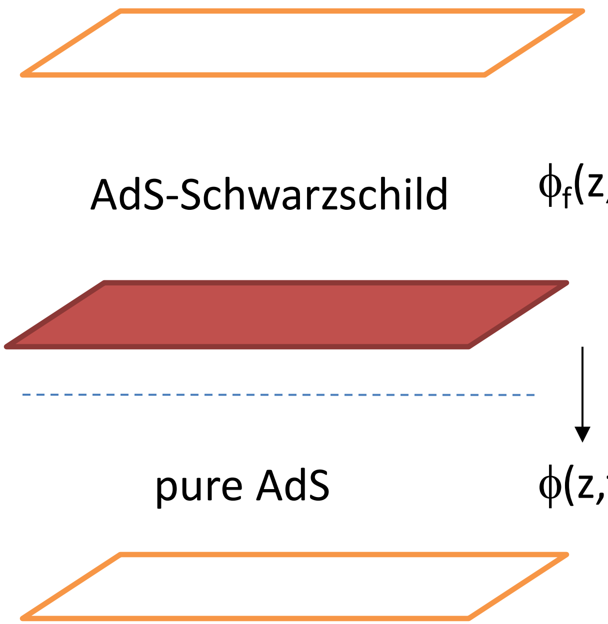
black red blue green brown

Far from equilibrium → Near equilibrium

R→0 for high frequency first.
Similar model, different probes: short distance scale thermalizes first (Balasubramanian et al 10)

Correlator beyond quasi-static approximation

Consider dilaton for simplicity



$\phi_f\ (z \rightarrow 0,t)=\delta(t-t')$

$\Box_f\ \phi_f\ =0$

$\phi_f(z,t_f,t')$

matching: continuity of the dilaton and its flux across the shell

$\phi(z,t,t')$

$\Box\ \phi\ =0$

$\phi\ (z \rightarrow \infty,t)$ ingoing wave for retarded correlator

Erdmenger, SL JHEP 12

PDE for massless bulk dilaton

$$G^R(t,t') = \int d^3x \theta(t-t')\langle[O(t,x),O(t',0)]\rangle$$
$$\neq G^R(t-t')$$

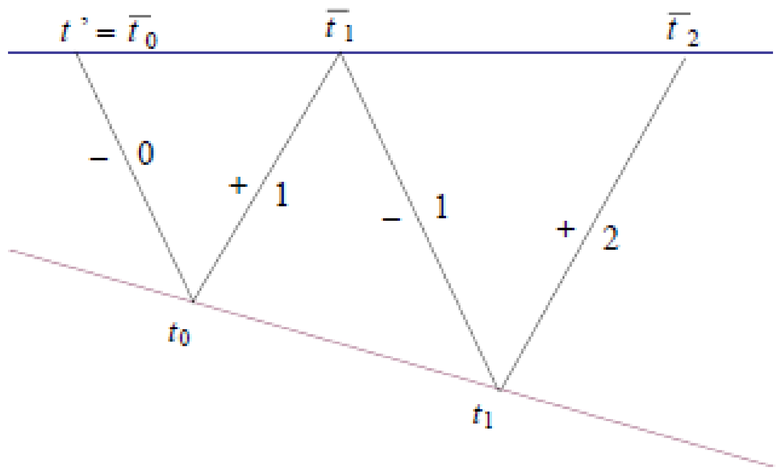
Retarded correlator for at q=0 does not respect time translational symmetry

Need to solve PDE for ϕ_f, ϕ to obtain $G^R(t,t')$. Initial conditions required.

In the high frequency limit (WKB), waves follow geometric optics in AdS. PDE reduces to ODE, thus initial conditions not essential.

Instead of solving the PDE numerically, we use a geometric optics method.

Illustration: divergence matching in pure AdS



Geometric optics in the bulk:

$\phi_f(t,t',z)$ singular along the segments (-,0), (+,1), (-,1) etc

positive/negative frequency contributions split in WKB limit

start with:

(-,0) ← singularities from bulk-boundary propagator in pure AdS

⋮

(+,1), (-,1), (+,2)… ← singularities due to bouncing on the shell and the AdS boundary

Erdmenger, Hoyos, SL JHEP 11

Illustration: divergence matching in pure AdS

Hologram: singularities in $G^R(t,t')$:

$$G^R(t,t') = \int d^3x\,\theta(t-t')\langle[O(t,x),O(t',0)]\rangle$$

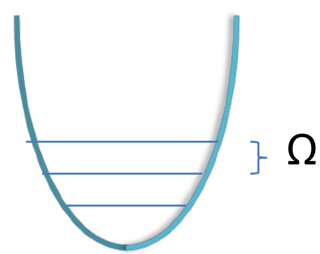
$$G^R(t \to \bar{t}_n, t') \sim \frac{B_n(-i)^{n-1}}{(-t+\bar{t}_n+i\varepsilon)^5} - \frac{B_n i^{n-1}}{(-t+\bar{t}_n-i\varepsilon)^5}$$

n=0 gives the light-cone singularity $t' = \bar{t}_0$

n>0 singularities contain information on the spectrum of the glue ball operator: existence of equally spaced Normal Mode in the spectrum

Elementary example: nonrelativistic harmonic oscillator

$$K(x,t,x',0) = \left[\frac{m\Omega}{2i\pi\sin\Omega t}\right]^{1/2} \exp\left\{\frac{im\Omega}{2\sin\Omega t}[(x^2+x'^2)\cos\Omega t - 2xx']\right\}$$



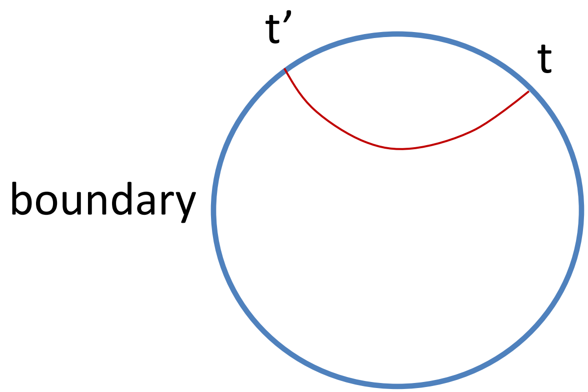
Singularities at $\Omega t = 2\pi k$

Relation with bulk-cone singularities conjecture

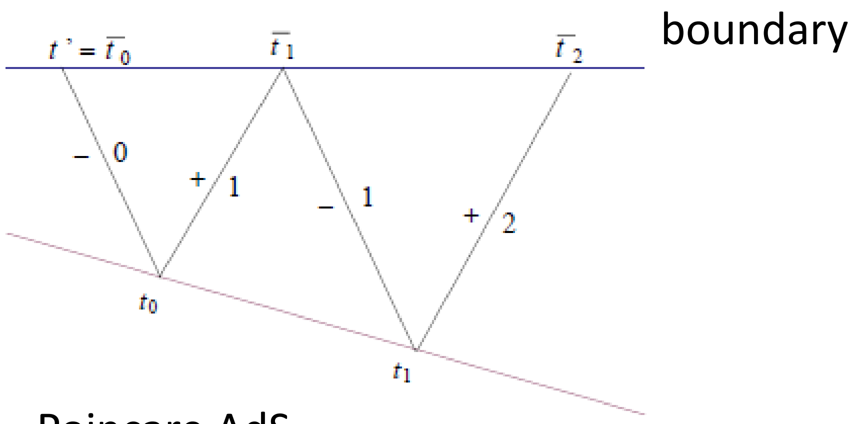
$$G^R(t \to \bar{t}_n, t') \sim \frac{B_n(-i)^{n-1}}{(-t+\bar{t}_n+i\varepsilon)^5} - \frac{B_n i^{n-1}}{(-t+\bar{t}_n-i\varepsilon)^5}$$

Bulk-cone singularities conjecuture:
Correlator becomes singular when two boundary points are connected by null geodesic in the bulk

Hubeny, Liu, Rangamani, JHEP 2007



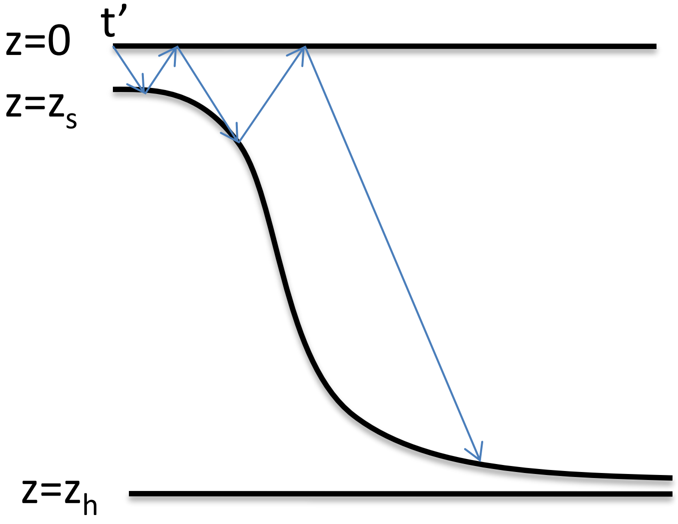
Global AdS



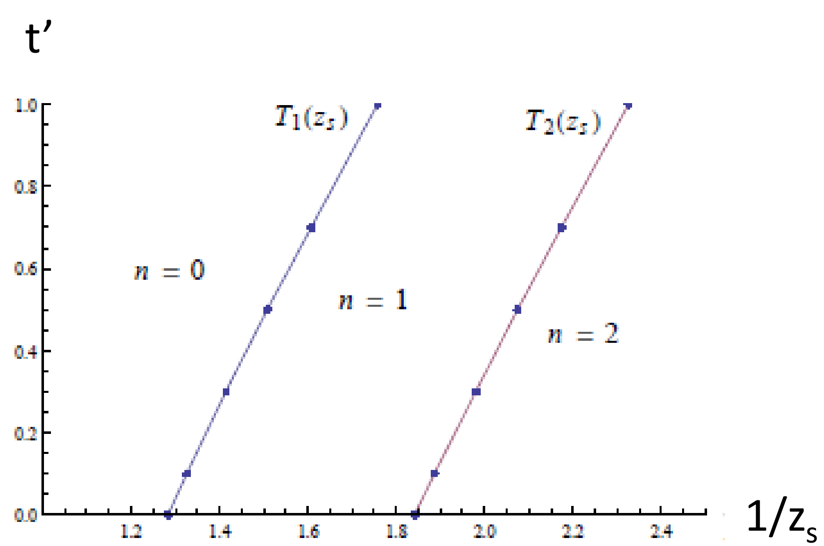
Poincare AdS

Light ray bouncing in collapse background

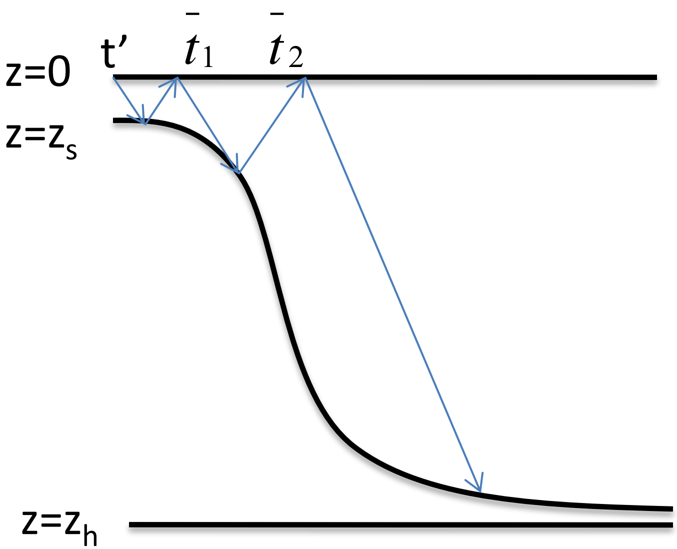
Expectation from geometric optics picture suggests singularities of $G^R(t,t')$ when the light ray starting off at t' returns to the boundary

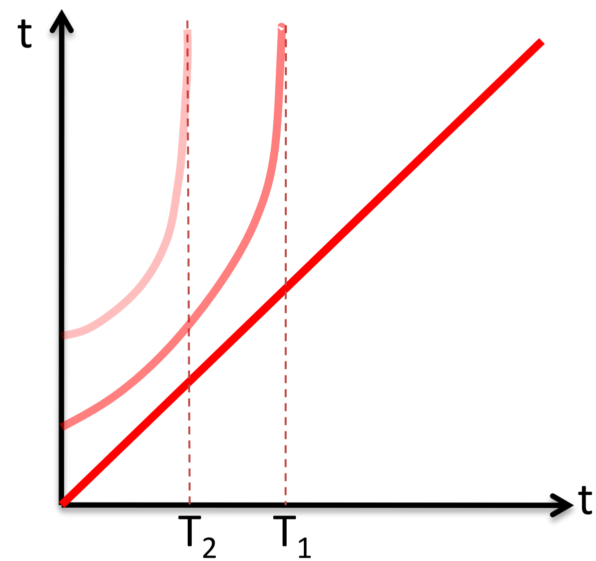


Only finite bouncings are possible:
The warping factor freeze both the shell and the light ray near horizon



Divergence matching in collapse background





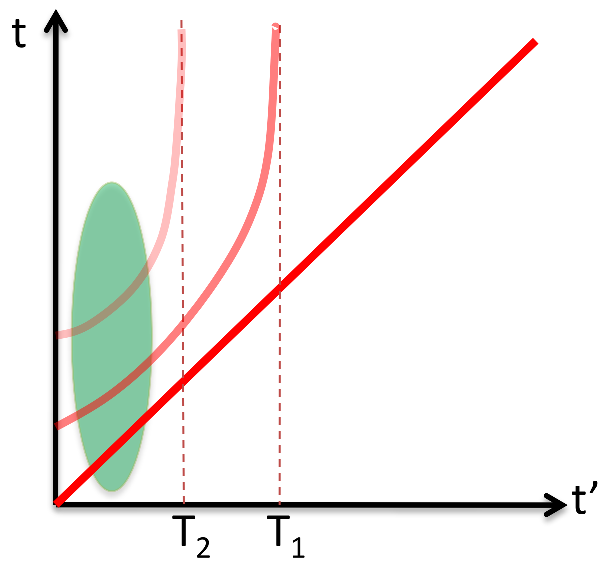
$$G^R(t \to \bar{t}_n) = \frac{A_n(-i)^{n-1}}{(-t+\bar{t}_n+i\varepsilon)^{5-n}} + \frac{A_n i^{n-1}}{(-t+\bar{t}_n-i\varepsilon)^{5-n}}$$

$\bar{t}_n \to +\infty$ as $t' \to T_n(z_s)$

Singularities weakened after each bouncing

"thermalization time" $$t_{th} = \frac{T_1(\pi T z_s)}{\pi T} \sim \frac{O(1)}{\pi T}$$

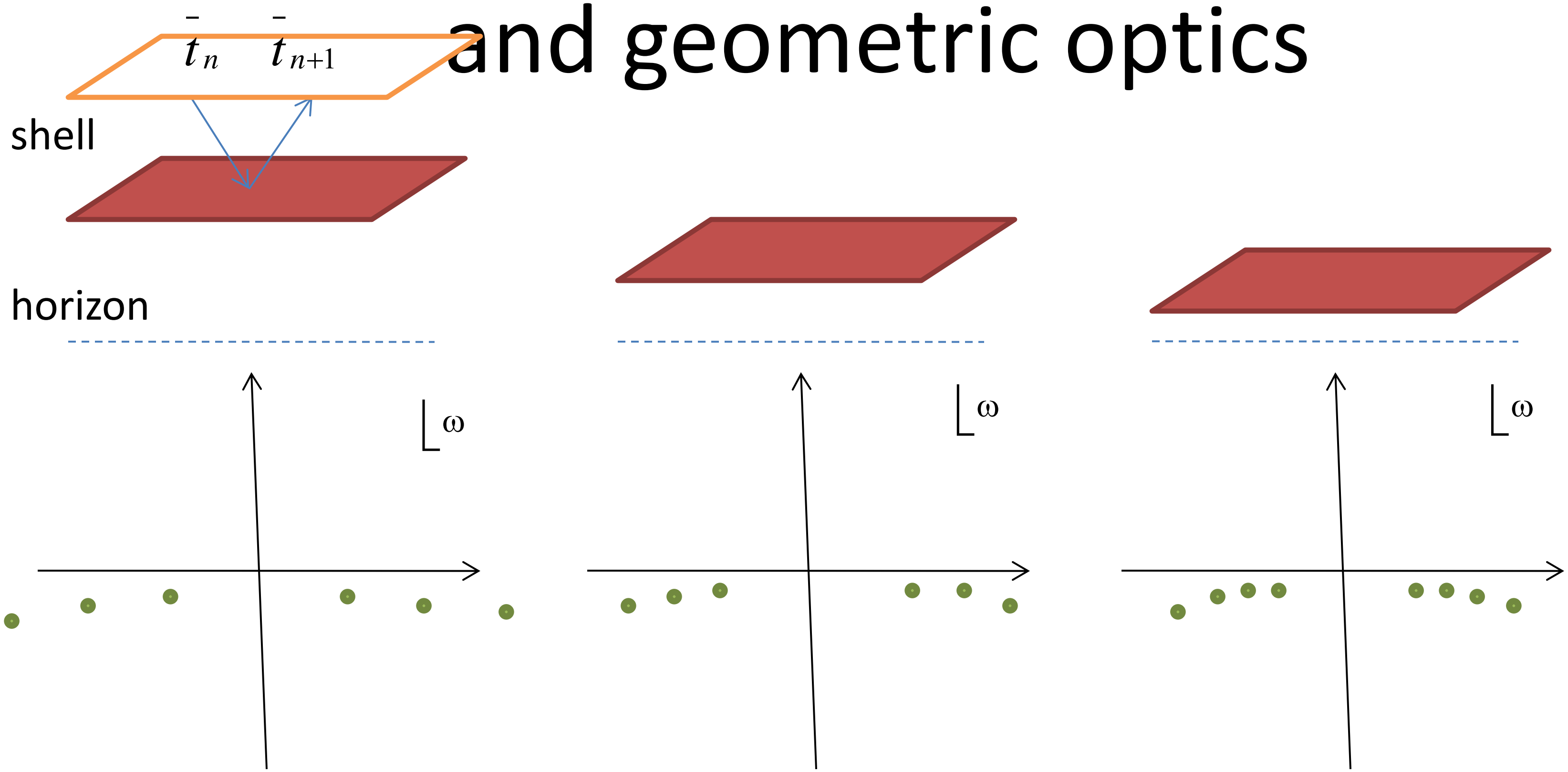
Comparison between the two approximations



Green region: quasi-static approximation valid
Red lines: geometric optics approximation valid
Other region: needs solving PDE

$$G^R(t,t') = \int d^3x\,\theta(t-t')\langle[O(t,x),O(t',0)]\rangle$$

Overlap region for both quasi-static and geometric optics

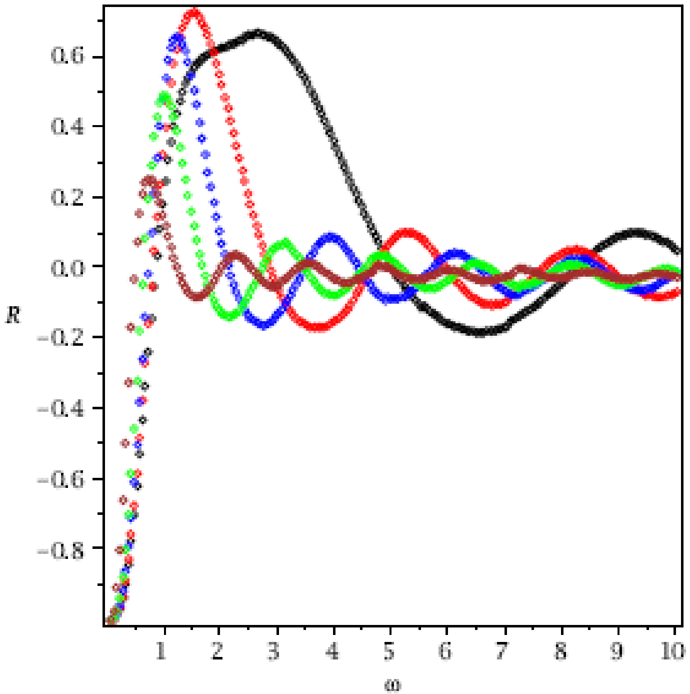


Approximate Normal Mode: Re(ω)>>Im(ω)

$$\Delta\omega \cdot (\bar{t}_{n+1} - \bar{t}_n) \approx 2\pi$$

See also Baier et al 1205.2998

Deviation from thermal spectral function for quasi-static state



black red blue green brown

Far from equlibrium

Near equlibrium

$$\chi = -2\,\mathrm{Im}\,G^R(\omega)$$ Spectral function

$$R = \frac{\chi - \chi_{thermal}}{\chi_{thermal}}$$

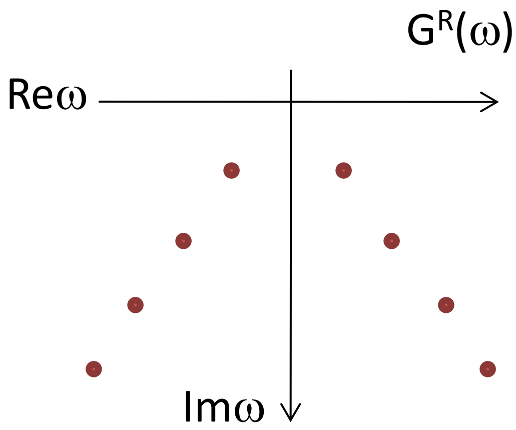
Spectral function for quasi-static state oscillate around thermal spectral function

Approximate Normal Mode responsible for the oscillation

$\Delta\omega$ shrinks as the shell is lowered toward the horizon

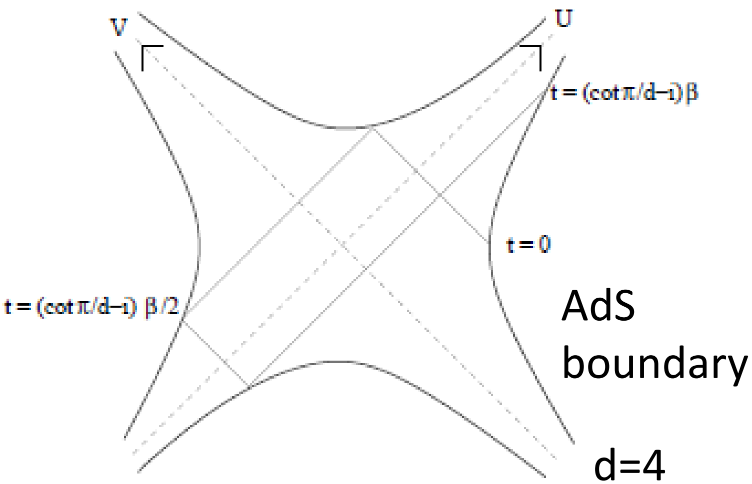
However, quasi-static approxiamtion breaks down very close to equilibrium!

Quasi-Normal Modes for thermal state



QNM for thermalized QGP obtained from scalar probe in AdS-Schwarzschild black hole

Quasi Normal Mode: Re(ω) ~ Im(ω)



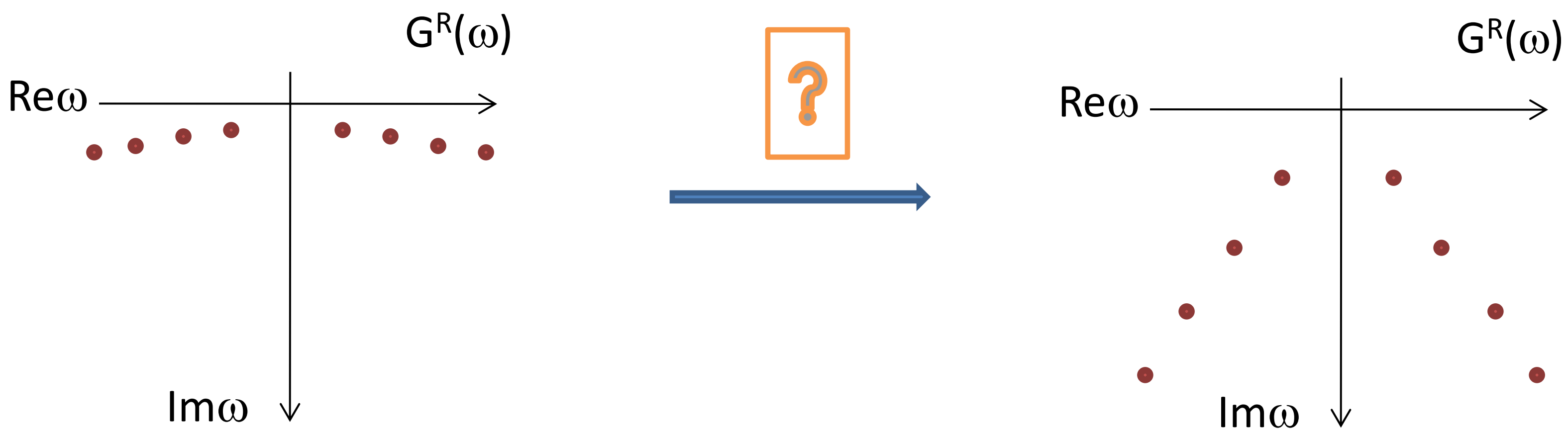
Bouncing light ray in Penrose diagram gives rise to complex time and therefore Quasi-Normal Modes

$$\Delta t = \frac{1}{2T}(1+i) \Rightarrow \Delta\omega = \frac{2\pi}{\Delta t} = 2\pi T(1-i)$$

$$\omega_n = 2\pi T(1-i)n$$

Amado & Hoyos JHEP 2008

Evolution of singularities in thermalization



Quasi-static approximation breaks down.
Need to go beyond the approximation to look at decay of low frequency mode.
Outside reach of geometric optics (high frequency approximation).
Need to solve full PDE

Summary

- We studied falling of stringy debris produced in the gravity dual of heavy ion collisions.
- The hologram of separating quark-anti-quark pair shows no jet like structure.
- We constructed a gravitational collapse model for thermalization.
- We studied two point function in the thermalization histoty, using both quasi-static approximation and geometric optics approximation. Results consistent with a top-down thermalization scenario.
- In quasi-static approximation: we have established the the singularities as from the contribution of approximate Normal Modes, which are also responsible for the oscillation in spectral function.
- In geometric optics approximation: finite singularities; singularities weakened at late time, and eventually disappear at late stage of thermalization.
- Regions not covered by the two approximations need to be explored numerically.

Thank you!