An Introduction to Higher Order Calculations in Perturbative QCD

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Overview of the Lectures

- Lecture I - Higher Order Calculations
  - What are they?
  - Why do we need them?
  - What are the ingredients and where do they come from?
  - Understanding and treating divergences
  - Examples from e^+e^- annihilation
- Lecture II - Examples of Higher Order Calculations
  - Parton Distribution Functions at higher order
  - Lepton Pair Production at higher order
  - Factorization scale dependence

- Lecture III - Hadronic Production of Jets, Hadrons, and Photons
  - Single inclusive cross sections
  - More complex observables and the need for Monte Carlo techniques
  - Overview of phase space slicing methods
- Lecture IV - Beyond Next-to-Leading-Order
  - When is NLO not enough?
  - Large logs and multiscale problems
  - Resummation techniques

Lecture III - Outline

- Overview of single particle inclusive calculations
  - General Form
  - Similarities to earlier examples
  - Basic properties of the results
- Extension to more complex observables
  - Problems with analytic calculations
  - Need for Monte Carlo methods
  - Methods to handle divergences
- Two-cutoff Phase Space Slicing Method
- Applications

Consider the forms of the lowest order calculations we have examined thus far

- DIS Structure Functions - Single integration over one PDF with a delta function $\delta(\eta - x)$
- Lepton Pair Production - Two integrations over a product of two PDFs with one delta function $\delta(Q^2 - x_a x_b S)$
- In both cases the end result of the NLO calculation was to add an $\mathcal{O}(\alpha_s)$ correction in addition to the delta function
- Now consider the hadroproduction process $A + B \rightarrow h + X$ where $A, B,$ and h are hadrons
- The basic cross section starts with an expression of the form

$$\begin{aligned} d\sigma(AB \rightarrow h + X) &= \frac{1}{2\hat{s}} \sum_{abcd} G_{a/A}(x_a, M_f^2)\, dx_a\, G_{b/B}(x_b, M_f^2)\, dx_b \\ & \alpha_s^2(\mu_r^2) \overline{\sum} |M_{ab\rightarrow cd}|^2\, D_{h/c}(z_c, M_f^2)\, dz_c\, dPS^{(2)} \end{aligned}$$

We can simplify this expression if we evaluate the two-particle phase space factor using the Mandelstam variables

$$\hat{s} = (p_a + p_b)^2 \quad \hat{t} = (p_a - p_c)^2 \quad \hat{u} = (p_b - p_c)^2$$

The two-body phase space factor is

$$dPS^{(2)} = \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{d^3 p_d}{(2\pi)^3 2E_d} (2\pi)^4 \delta(p_a + p_b - p_c - p_d)$$

Exercise: Show that $\frac{d^3 p_d}{2E_d} = d^4 p_d \delta(p_d^2)$ and that $\delta(p_d^2) = \delta(\hat{s} + \hat{t} + \hat{u})$ for massless partons and hence that

$$dPS^{(2)} = \frac{1}{8\pi^2} \frac{d^3 p_c}{E_c} \delta(\hat{s} + \hat{t} + \hat{u})$$

Things will further simplify if we introduce two dimensionaless variables which vary between zero and one:

$$w = -\frac{\hat{s}+\hat{t}}{\hat{u}} \qquad v = 1 + \frac{\hat{t}}{\hat{s}}$$

Finally, the parton and final state hadron momenta by are related by $p_h = z_c p_c$

Exercise: Show that the invariant hadronic cross section can be written as

$$\begin{aligned} E_h \frac{d^3\sigma}{dp_h^3} \quad &= \quad \sum_{abcd} \int dx_a \, dx_b \, \frac{dz_c}{z_c^2} G_{a/A}(x_a, M_f^2) G_{b/B}(x_b, M_f^2) D_{h/c}(z_c, M_f^2) \\ & \quad \frac{1}{16\pi^2 \hat{s}^2 v} \overline{\sum} |M(ab \to cd)|^2 \delta(1-w) \end{aligned}$$

(Why did I call this the invariant cross section?)

We have the desired form of two PDFs times a FF times a squared amplitude and factors times a delta function

Now, let's look at the form of the NLO corrections

By now, we can anticipate what needs to be done

- Use the two-loop running coupling
- Use NLO PDFs
- Use $2 \to 3$ tree-level matrix elements and $2 \to 2$ loop corrections
- Use three particle phase space for the tree graphs
- Use dimensional regularization
- Factorize the collinear singularities associated with the PDFs and FFs
- This will result in an $\mathcal{O}(\alpha_S)$ correction to the preceding expression for the invariant cross section

Three-body Phase Space

$$dPS^{(3)} = \frac{d^3p_c}{(2\pi)^3 2E_c}\frac{d^3p_d}{(2\pi)^3 2E_d}\frac{d^3p_e}{(2\pi)^3 2E_e}(2\pi)^4\delta(p_a + p_b - p_c - p_d - p_e)$$

- An easy way to understand this is to think of the $p_d p_e$ system as a two-body system with a 4-vector $p_{de} = p_d + p_e$
- This two-body system then has the standard two-body phase space factor in terms of the polar and azimuthal angles θ_1 and θ_2 in the $d + e$ cm frame.
- Then we tack on the p_c factor and evaluate it using two-body kinematics for $p_a + p_b \to p_c + p_{de}$
- Now, $p_{de}^2 \neq 0$, so the Mandelstam variables defined before now satisfy $\hat{s} + \hat{t} + \hat{u} = p_{de}^2$ where the Mandelstam variables are formed from $p_a, p_b,$ and p_c

- The net effect is that w is no longer equal to one as it was for the lowest order terms - it approaches one when partons d and e are parallel giving $p_{de}^2 = 0$
- The three-body phase space factor in n dimensions is given by

$$dPS^{(3)} = \frac{\hat{s}}{2^8\pi^4\Gamma(1-2\epsilon)}\left(\frac{\hat{s}}{4\pi}\right)^{-\epsilon} v^{1-2\epsilon}(1-v)^{-\epsilon}w^{-\epsilon}(1-w)^{-\epsilon}\sin^{1-2\epsilon}\theta_1\sin^{-2\epsilon}\theta_2$$

- We will now be integrating over w and we see a factor $(1-w)^{-\epsilon}$ which will regulate divergences at $w=1$
- Why do we expect divergences at $w=1$? This is the two-body limit of three-body phase space. Either two partons have become collinear, thereby behaving as one massless parton (collinear limit), or one of the partons has zero momentum (soft limit)
- Either way, we expect to see factors like $(1-w)^{-1}$ coming from the squared $2 \to 3$ matrix elements

- We also anticipate that the expansion in terms of plus distributions of factors like $(1-w)^{-1-\epsilon}$ will generate terms involving

$$\delta(1-w), \frac{1}{(1-w)_+}, \text{ and } \left(\frac{\ln(1-w)}{1-w)}\right)_+$$

- Of course, I picked singularities where partons d and e were collinear or one was soft. Clearly there are regions where other pairs become collinear. If c denotes the parton which fragments into the hadron, then c can never be soft.
- Thus, there are other plus distributions than the ones I listed, but you get the idea.
- At the end of the day, the basic structure of the answer looks like

$$\begin{aligned}\delta(1-w) \quad &\rightarrow \quad \delta(1-w) + \\ &\frac{\alpha_s}{2\pi}\left[f_\delta \delta(1-w) + f_1 \frac{1}{(1-w)_+} + f_2 \left(\frac{\ln(1-w)}{1-w}\right)_+ + \cdots + f_f\right]\end{aligned}$$

- Here the f coefficients are functions of v and w, the dots indicate other plus regulators, and f_f represents all the terms which do not involve plus regulators or delta functions
- Note: The contributions from the loop diagrams contribute to f_δ since they have the same kinematics as the lowest order terms
- The actual calculation will be a bit more complex, since there are many subprocesses to consider for a typical high-p_T process, but this is the general structure for each subprocess
- Note: some contributions won't have a delta function piece since they don't have a counterpart in the lowest order (think of the gluon term in DIS)
- Now, where would we expect this type of correction to be useful and what are the properties of the corrections?

- Generally, the high-p_T calculation outlined above is appropriate for problems where there is one large scale
- This means that $\hat{s} \sim \hat{t} \sim \hat{u}$
- In terms of the kinematic parameters of the observed hadron, this means that the rapidity is not near the edge of phase space and that the transverse momentum is large, *i.e.*, $x_T = \frac{2p_T}{\sqrt{S}}$ is neither near one nor near zero
- In this region the NLO calculation will generally provide
  - Reduced scale dependence
  - A modest correction to the normalization of the lowest order result
- There may be some changes in the p_T and y distributions due to the presence of new subprocesses
- Note that in this example we have integrated over both of the recoiling partons. This smooths out regions where there might otherwise have been large corrections.
- Next, let's look at how the reduction in scale dependence actually comes about and how we might use this to our advantage

Scale dependence

- Consider a highly simplified example of jet production in hadron-hadron scattering at large enough values of x_T that only valence quark scattering subprocesses need be considered.
- Denote the lowest order result for the invariant cross section by

$$E\frac{d^3\sigma}{dp^3} \equiv \sigma = \alpha_s^2(\mu)\hat{\sigma}_B \otimes q(M) \otimes q(M)$$

- Here $\hat{\sigma}_B$ denotes the lowest order parton-parton scattering cross section while $q(M)$ denotes a quark PDF with factorization scale M
- I have separated out the running coupling which is evaluated at a renormalization scale μ
- $\otimes$ denotes a convolution

$$f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

- With this same notation the NLO calculation will have the form

$$\begin{aligned}\sigma &= \alpha_s^2(\mu)\hat{\sigma}_B \otimes q(M) \otimes q(M) \\ &+ 2b\alpha_s^3(\mu) \ln \frac{\mu^2}{p_T^2} \hat{\sigma}_B \otimes q(M) \otimes q(M) \\ &+ 2\frac{\alpha_s^3(\mu)}{2\pi} \ln \frac{p_T^2}{M^2} P_{qq} \otimes q(M) \otimes q(M) \\ &+ \alpha_s^3(\mu) K \otimes q(M) \otimes q(M)\end{aligned}$$

- I have separated out the parts of the NLO correction which contain explicit logs of μ or of M and have normalized then using p_T
- K denotes the remainder of the NLO correction

Recall that

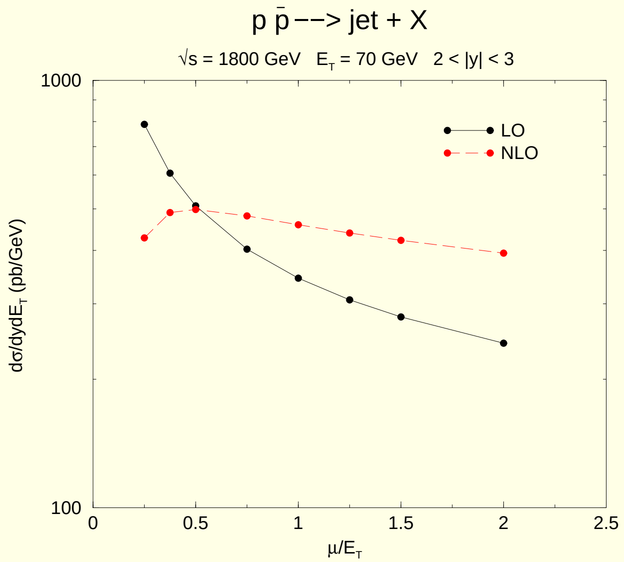
$$\mu^2 \frac{\partial \alpha_s(\mu)}{\partial \mu^2} = -b\alpha_s^2 + \ldots$$

and that the nonsinglet PDF satisfies

$$M^2 \frac{\partial q(x, M)}{\partial M^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes q(M)$$

- Now, calculate $\mu^2 \frac{\partial \sigma}{\partial \mu^2}$
- The derivative of the first line gives a contribution which cancels a piece of the derivative of the second line; the remaining derivatives of the second, third, and fourth lines all give contributions of $\mathcal{O}(\alpha_s^4)$
- The μ dependence is thus zero to $\mathcal{O}(\alpha_s^3)$ (Exercise: Fill in the steps to show this)

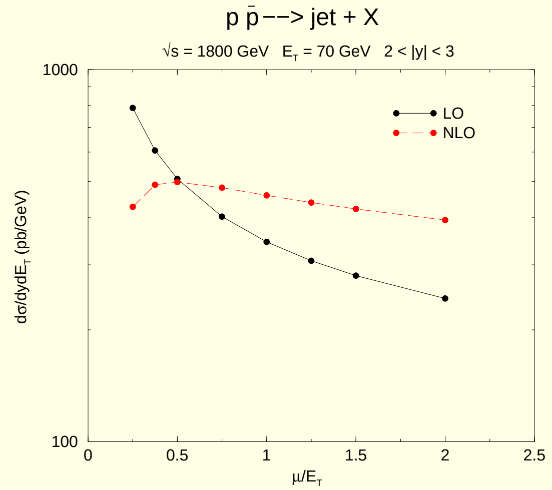
- Now, calculate $M^2 \frac{\partial \sigma}{\partial M^2}$
- Again, the derivative of the first line cancels a portion of the derivative of the third and the remaining derivatives give results of $\mathcal{O}(\alpha_s^4)$ (Exercise: Fill in the steps to show this)
- Both the renormalization and factorization scale dependences cancel to the order calculated, although there is still residual scale dependence due to higher order corrections
- The following plot shows the type of behavior which is typical of LO and NLO calculations



Understanding the scale dependences

- To simplify the discussion, consider the situation where $\mu = M$, as in the previous plot
- For the lowest order calculation we understand that increasing μ causes the running coupling to decrease
- In the region of $x \gtrsim .1$ an increase of M also causes the PDFs to decrease - this is the region relevant for our high-p_T jet example
- Thus, the LO calculation is a monotonically decreasing function of the scale
- For the full NLO calculation, the first line (lowest order result) and the last line (residual NLO result) both have the same type of monotonically decreasing behavior as the scale increases
- The $\ln \frac{\mu^2}{p_T^2}$ factor in the second line causes this contribution to be negative for $\mu < p_T$ and to be positive once μ exceeds p_T
- For the third line, recall that the convolution with the splitting function gives a negative contribution in the region of interest since the slope of the scaling violations is negative there

- Thus, for $M < p_T$ the third line is negative and it turns positive for $M > p_T$
- The explicit logs in lines 2 and 3 thus cause the NLO curve to be below the LO curve if $\mu = M < p_T$ and to be above it if the scales are greater than p_T, as shown in the plot



- Note that this is an approximate argument and that there can be exceptions to it caused, for example by new channels opening in higher order
- The exact crossover point depends on the relative sizes of the contributions from each of the four lines

“Which scale is best?”

There are various methods for guestimating the choice of scale in NLO processes. Here are some examples:

- Principle of Minimal Sensitivity
  - An exact calculation would have no scale dependence - perturbative calculations are incomplete
  - The PMS scheme enforces $\frac{\partial \sigma}{\partial \mu} = \frac{\partial \sigma}{\partial M} = 0$
  - In my example the full dependence on μ was displayed so one can solve for the value of μ which makes the derivative zero - not the same as having it be zero to $\mathcal{O}(\alpha_s^3)$
  - Similarly, can numerically solve for the value of M which forces $\frac{\partial \sigma}{\partial M}$ to be zero
  - For each kinematic point the plot of μ versus M gives a saddle point structure and one can read off the correct values for both scales

- Could also use a "1-scale" PMS scheme - then one can read off the optimal scale from plots like the one shown previously which suggested $\mu = M \approx p_T/2$
- Method of Fastest Apparent Convergence
  - Choose the scale such that NLO and LO calculations are equal
  - All the higher order corrections are effectively absorbed into the logs of the scales

There is no unique prescription for choosing the scales

- When the higher order corrections are under control, both schemes give similar results of the order of the single large scale in the process
- In the plot shown earlier both schemes would suggest the choice of $p_T/2$ which is close to the "natural" choice of p_T

The ratio of the NLO and LO curves is just the K factor. It is obviously very scale dependent. For the FAC scheme the K factor is *defined* to be 1!

More complicated observables

For the single particle cross section it is easy to calculate both the p_T and y distributions. However, there are other interesting observables

- Jets - one needs to be able to form jets of according to some jet definition which may not be easy to express in terms of partonic variables
- One might wish to examine joint distributions involving more than one particle
- Classic example - one might wish to calculate or measure the angular distribution of the scattered partons in their center of mass frame. With $2 \to 3$ subprocesses, how do you define this?
- One might wish to place cuts (kinematic constraints) on the final state particles. Sometimes this is easy (cuts in p_T or y)
- Sometimes the Jacobian between the experimentally observed variables and the parton level variables can not be easily calculated
- Suggests using a Monte Carlo formalism so that the cuts can be made on an event-by-event basis
- But what about the divergent terms?

Next-to-Leading-Order Calculations – Recap

- Ingredients
  - $2 \to 2$ $\mathcal{O}(\alpha_s^2)$ subprocesses, *e.g.*, $qq \to qq, qg \to qg,$ and $gg \to gg$
  - $\mathcal{O}(\alpha_s^3)$ one-loop corrections to $2 \to 2$ subprocesses
  - $2 \to 3$ subprocesses such as $qq \to qqg$, etc
- $\mathcal{O}(\alpha_s^3)$ terms have singular regions corresponding to soft gluons and/or collinear partons
- Need a method to handle such singularities
- Observables involve many kinematic variables since we are interested in going beyond the single particle case
- Jacobians from parton variables to hadron variables are complex
- Suggests using a Monte Carlo approach, but one which allows the singularities to be properly treated

Phase Space Slicing Monte Carlo

- See B. Harris and J.F. Owens hep-ph/0102128
- Work in n=4-2ϵ dimensions using dimensional regularization
- Notation:
  - At the parton level: $p_1 + p_2 \rightarrow p_3 + p_4 + p_5$
  - Let $s_{ij} = (p_i + p_j)^2$ and $t_{ij} = (p_i - p_j)^2$
- Partition $2 \rightarrow 3$ phase space into three regions
  1. Soft: gluon energy $E_g < \delta_s \sqrt{s_{12}}/2$
  2. Collinear: s_{ij} or $|t_{ij}| < \delta_c s_{12}$
  3. Finite: everything else

- In soft region use the soft gluon approximation to generate a simple expression for the squared matrix element which can be integrated by hand
- In the collinear region use the leading pole approximation to generate a simple expression which can be integrated by hand.
- Resulting expressions have explicit poles from soft and collinear singularities
- Factorize initial and final state mass singularities and absorb into the fragmentation and distribution functions
- Add soft and collinear integrated results to the $2 \to 2$ contributions – singularities cancel
- Generate finite region contributions in 4 dimensions using usual Monte Carlo techniques
- End results is a set of two-body weights and a set of three-body weights.
- Both are finite and both depend on the cutoffs δ_s and δ_c
- Cutoff dependence cancels for sufficiently small cutoffs when the two sets of weights are added at the histogramming stage

Simple Example

Consider the integral of a quantity which has a pole at $x = 0$. Using dimensional regularization, one has an integral of the form

$$F = \int_0^1 dx\, x^{-1-\epsilon} f(x).$$

For x very near zero, approximate $f(x)$ by $f(0)$ yielding

$$F \approx f(0) \int_0^\delta dx\, x^{-1-\epsilon} + \int_\delta^1 dx\, x^{-1-\epsilon} f(x)$$

The first integral can be done analytically. The second is finite and can be evaluated with $\epsilon = 0$.

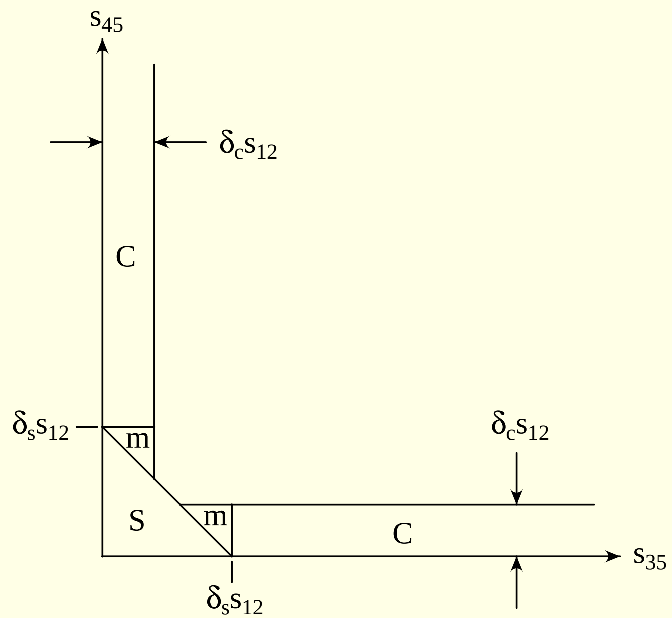
$$F \approx -\frac{f(0)}{\epsilon} + f(0) \log \delta + \int_\delta^1 dx\, \frac{f(x)}{x}.$$

The second integral can be done numerically. The dependence on the cutoff δ cancels for sufficiently small values of δ

Another Simple Example

Consider the example from Lecture I of the total cross section for $e^+e^- \rightarrow hadrons$. The complete first order QCD correction is simply $\frac{\alpha_s}{\pi}$.

- Phase space can be written in terms of two variables. It is convenient to choose these to be s_{35} and s_{45}.



- This sketch shows the soft, hard collinear, and finite regions of phase space

- The 1-loop virtual corrections lie at the origin in the lower left and are included in the soft region
- In the soft region the squared matrix element takes on a relatively simple form which may be integrated to yield

$$d\sigma_{\mathrm{S}} = d\sigma^0 \left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{s_{12}}\right)^{\epsilon}\right]\left(\frac{A_2^s}{\epsilon^2}+\frac{A_1^s}{\epsilon}+A_0^s\right)$$

with

$$\begin{aligned} A_2^s &= 2C_F \\ A_1^s &= -4C_F \ln\delta_s \\ A_0^s &= 4C_F \ln^2\delta_s \end{aligned}$$

- The final state hard collinear contribution can be simplified in the collinear region and easily integrated to yield

$$d\sigma_{\mathrm{HC}}^{q\to qg} = d\sigma^0 \left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{s_{12}}\right)^{\epsilon}\right]\left(\frac{A_1^{q\to qg}}{\epsilon}+A_0^{q\to qg}\right)$$

with

$$\begin{aligned} A_1^{q\to qg} &= C_F\,(3/2 + 2\ln\delta_s) \\ A_0^{q\to qg} &= C_F\left[7/2 - \pi^2/3 - \ln^2\delta_s - \ln\delta_c\,(3/2 + 2\ln\delta_s)\right] \end{aligned}$$

- The virtual contribution is given by

$$d\sigma_{\rm V} = d\sigma^0\left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{s_{12}}\right)^{\epsilon}\right]\left(\frac{A_2^v}{\epsilon^2} + \frac{A_1^v}{\epsilon} + A_0^v\right)$$

  with

$$\begin{aligned} A_2^v &= -2C_F \\ A_1^v &= -3C_F \\ A_0^v &= -2C_F(4 - \pi^2/3) \end{aligned}$$

- The full two-body weight is given by the sum $d\sigma_{\rm S} + d\sigma_{\rm V} + 2d\sigma_{\rm HC}^{q\to qg}$. The factor of two occurs since there are two quark legs, either of which can emit a gluon.

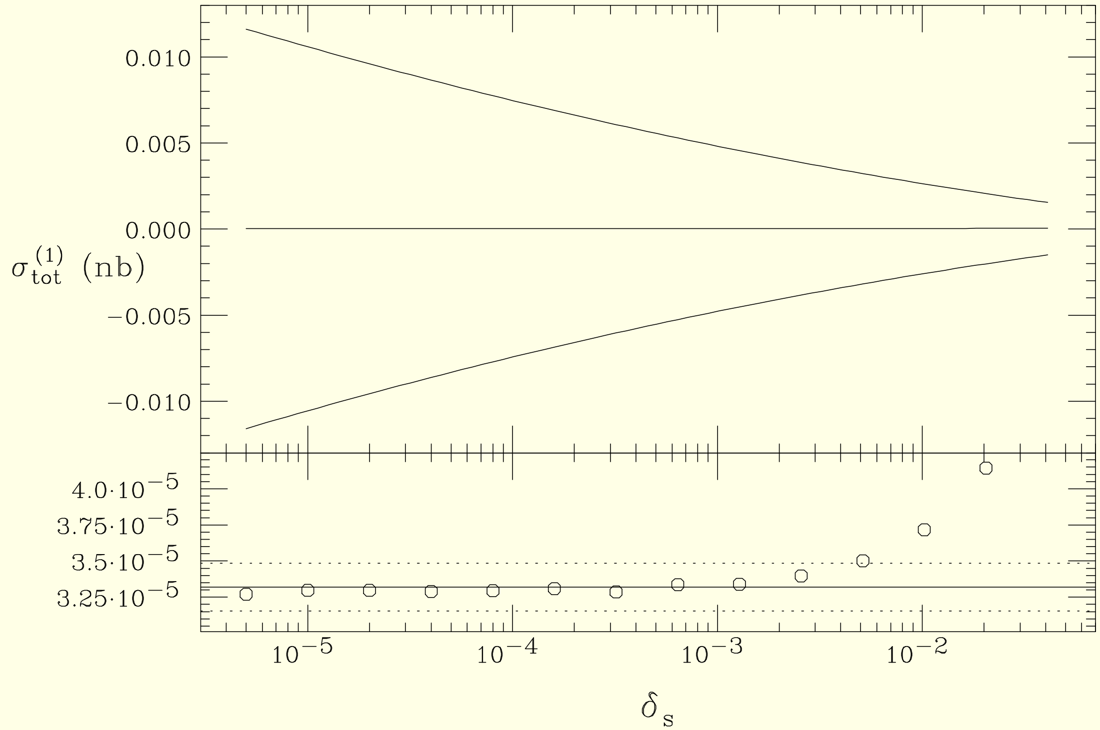
- At this point we have a finite result since $A_2^s + A_2^v$ and $A_1^s + A_1^v + 2A_1^{q\to qg}$ both separately add up to zero
- The finite two-body weight is given by

$$\sigma^{(2)} = \int d\sigma_0 \left(\frac{\alpha_s}{2\pi}\right) \left(A_0^s + A_0^v + 2A_0^{q\to qg}\right)$$

while the three-body contribution is given by

$$\sigma^{(3)} = \sigma_{H\overline{C}} = \frac{1}{2s_{12}} \int_{H\overline{C}} \overline{\sum} |M_3|^2 dPS^3$$

- The final result is shown in the following figure as a positive three-body weight, a negative two-body weight and the finite sum

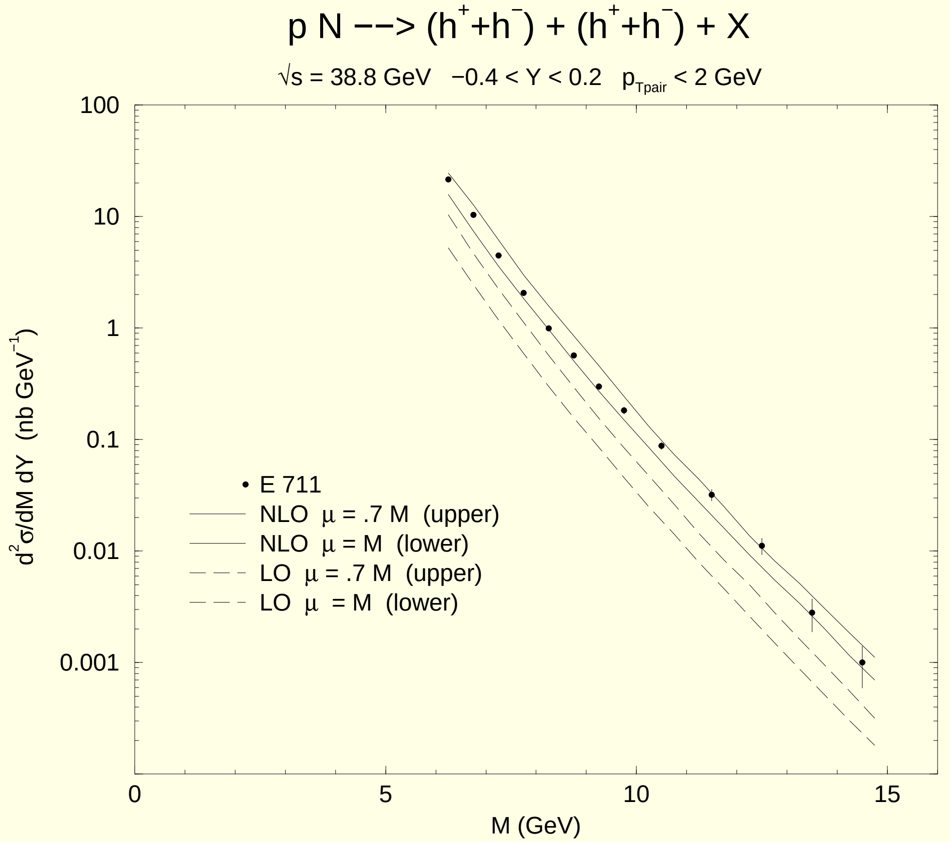


- The results are plotted versus δ_s with $\delta_c = \delta_s/300$
- The solid horizontal line is the exact result
- The method converges nicely, provided that the cut-offs are small enough
- Note: the small triangular regions denoted by m in the phase space figure are not included in the calculation. Their contribution can be included, but it is of order δ_c/δ_s and is negligible provided that $\delta_c << \delta_s$

Hadron Pair Production

- Two cutoff phase space slicing technique was originally motivated by the need for a NLO calculation for Fermilab experiment E-711
- NLO calculation needed in order to reduce the scale dependence
  - Two powers of α_s
  - Two PDFs
  - Two FFs
  - In the typical kinematic region all six factors cause the cross section to decrease as the scale is increased
- Would like to examine the parton-parton scattering process angular distribution
- Difficult to map the partonic variables to the observed variables which suggests a Monte Carlo approach

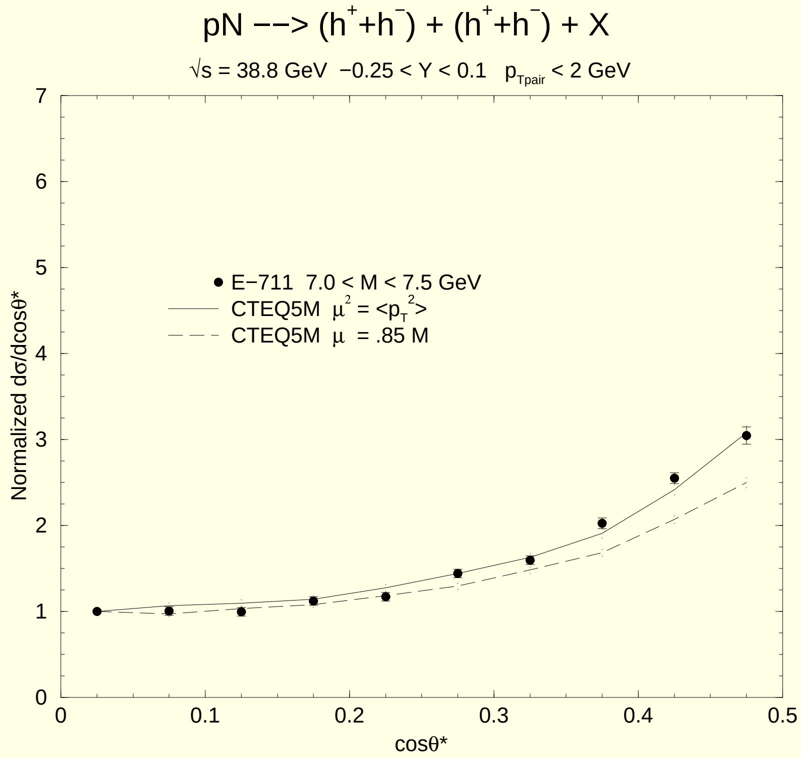
- Kinematic variables for the dihadron system
  - Mass of the dihadron system M
  - Rapidity of the hadron pair Y
  - Cosine of the scattering angle in the parton-parton system $\cos\theta^*$
  - Transverse momentum of the dihadron system $p_{T pair}$
  - Event sample may also involve cuts being placed on the transverse momenta and rapidities of the individual hadrons
- Technique is the same as outlined previously
- Two-body weight consists of
  - Lowest order contributions
  - virtual contributions
  - Collinear contributions
  - Soft contributions
- Three-body weights consist of all the finite $2 \rightarrow 3$ contributions
- Generate both sets of weights and add together at the histogramming stage

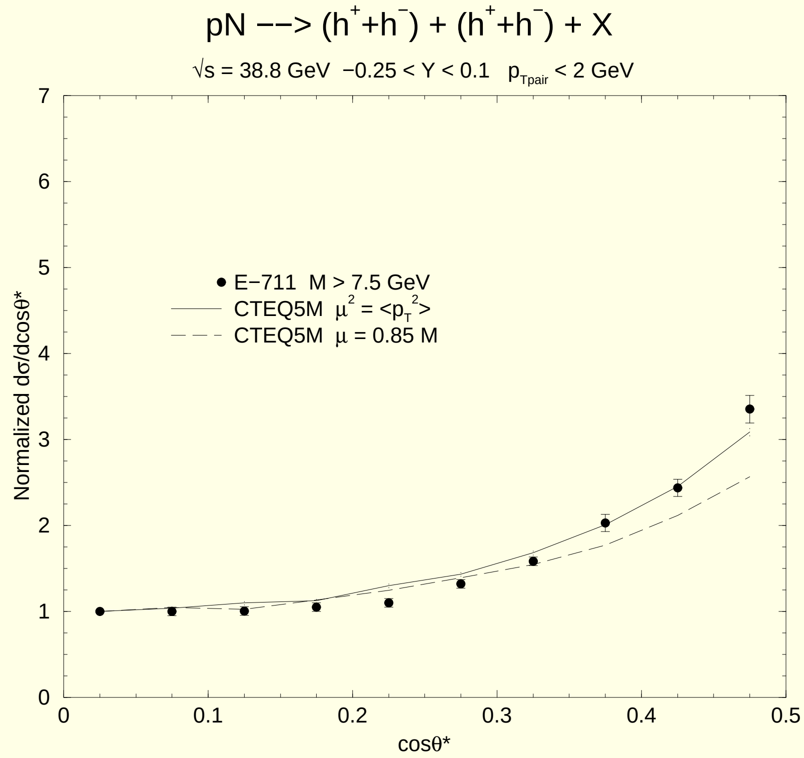


- NLO results significantly higher than LO results
- Scale dependence reduced, but still significant

Angular distributions

- Boost to frame where the two hadron system has zero longitudinal momentum
- Calculate cosine of the angle between each hadron and the beam direction
- Won't be the same since the two hadrons aren't exactly back to back (transverse momenta need not be the same in this frame because of fragmentation effects, for example)
- Use average of the two values
- Scale choice: for $d\sigma/dM$ only have one variable with the dimension of mass to use for the scale - M
- For $d\sigma/d\cos\theta^*$ at fixed M, one could choose M again or something like $M\sin\theta^*$ which is proportional to the transverse momentum of the individual hadrons
- Use two choices: M and the average of the two squared p_T values



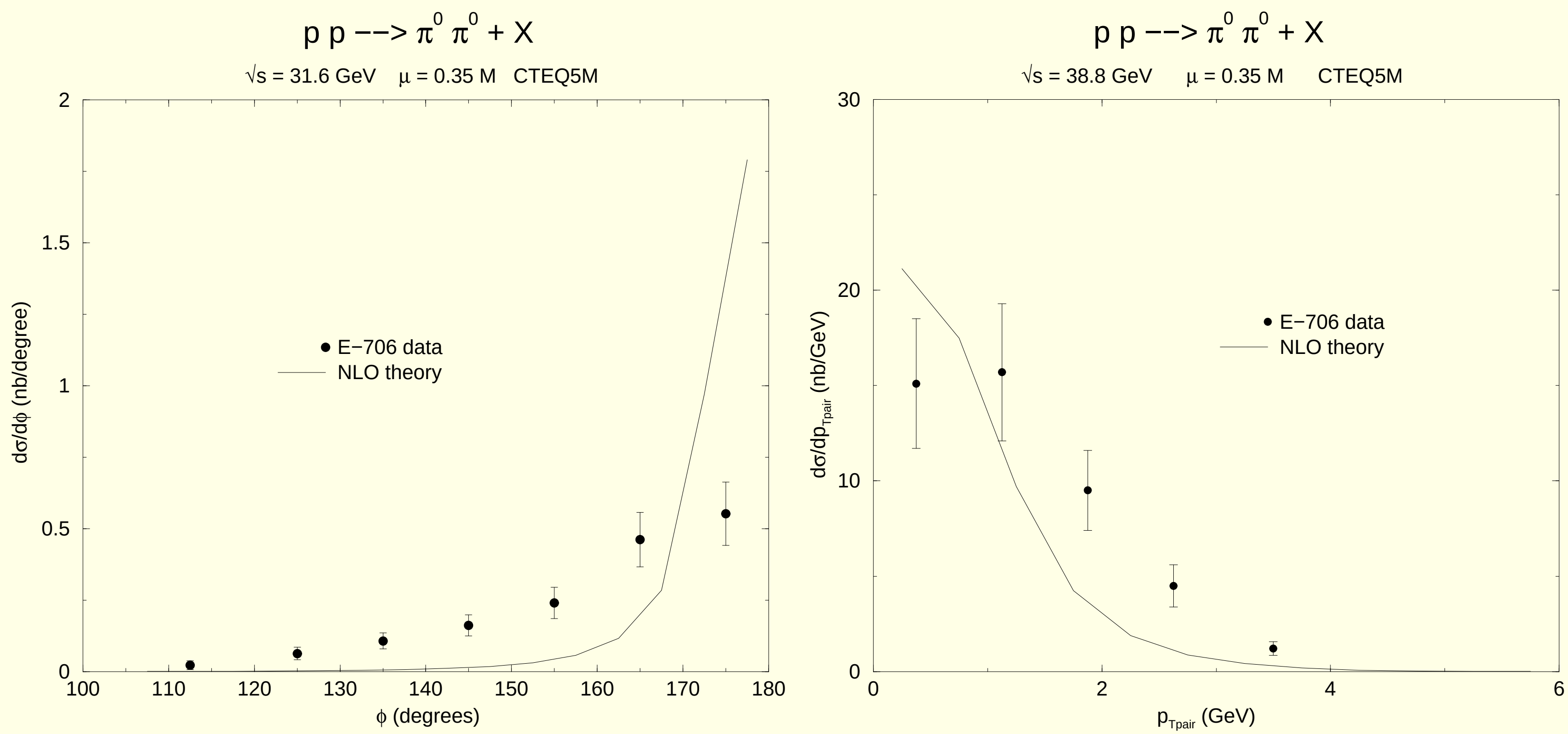


Choice of $< p_T^2 >$ gives a steeper curve, in better agreement with the data. At fixed M larger $\cos\theta^*$ gives a smaller scale which increases the cross section.

When NLO Doesn't Do the Job

Consider the distribution of the azimuthal angle between the two observed hadrons. The kinematics appropriate for the lowest order calculation gives this as a delta function at $\phi = \pi$. The NLO calculation gives an additional parton in the final state

- Can calculate the *tail* of the distribution far from $\phi = \pi$
- As ϕ approaches π the additional parton is forced either to be collinear or soft $\Rightarrow$ divergences at $\phi = \pi$
- Calculation breaks down at this point $\Rightarrow$ need additional partons so that the constraint $\phi = \pi$ can be satisfied without forcing the partons to be soft or collinear
- Need to use resummation techniques
- Similar comments pertain to the $p_{T pair}$ distribution



Theoretical distributions with just one additional parton are too narrow

- If experimental cuts are placed on ϕ or on the p_T of the pair, then too much of the theory will be included compared to the data
- Must use large kinematic regions in these variables if the theory and experiment are to agree
- Both distributions are δ-functions at lowest order
- Correct description requires the use of the appropriate resummation formalism
- Will get misleading results if cuts are placed on a distribution which is not well described by the theory
- If different experiments use different cuts on these variables in order to define their event sample, then it may appear that the theory is in better agreement with one than the other
- Better to integrate over the variable in question if the acceptance of the experiment allows it

Classic situation

- Some experimenters place tight cuts on ϕ and on $p_{T pair}$ in order to make the events "look more like the lowest order predictions"
- At lowest order these cuts include all the lowest order theory, but only a small fraction of the data
- At NLO, the weights at the two-body boundary are negative - you have to integrate over the $2 \rightarrow 3$ events to get a positive result

Bad things happen when tight cuts are used to define a "lowest order" sample

Other Applications

- Processes where phase space slicing type methods have been used include
  - γ, γ + jet, and γ + hadron processes
  - Jet and dijet production
  - Single hadron and dihadron production
  - Vector boson and diboson production
  - and many others
- Method provides an improved treatment of isolation cuts in photon processes
- Method has proven to be useful for dihadron and photon-hadron tomography in high energy nuclear collisions
- Method is useful for jet studies, as different jet algorithms can be implemented easily at the parton level

Summary

- Have seen how the NLO formalism works for single particle production in hadron-hadron processes
- Have looked at more complicated observables and correlations
- Have seen how Phase Space Slicing techniques allows one to use flexible Monte Carlo techniques while ensuring that the singularities are properly factorized
- Have examined some examples and applications of this technique
- Have looked at some cases where NLO calculations are not sufficient to describe the data and more terms are required