Parton Distribution Functions and Global Fitting

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Preliminaries

Issues related to PDFs and global fitting have been addressed at several of the previous CTEQ Summer Schools. Many of the lectures can be found at the CTEQ web site: www.cteq.org.

- 2006 - Mandy Cooper-Sarkar: excellent coverage of data sets and various issues of global fitting
- 2005 - Dan Stump: Excellent discussion on PDF uncertainties
- 2004 - Wally Melnitchouk - discussion of low-energy models of the nucleon, nuclear corrections, measurements of d/u
- 2003 - Alan Martin: good overview of issues in global fits including low-x, effects of NNLO, and PDF errors
- 2002 - Walter Giele: probability theory and alternative means of global fitting, using PDF sets
- 2001 - Robert Thorne: Overview of global fits and PDF uncertainties
- 2000 - James Stirling: Emphasis on PDFs in DIS

Note: M, S, and T of MRST have all lectured at our schools.

Goals for these lectures

The previously cited lectures give excellent descriptions of various aspects of PDFs and issues related to global fitting. My goals are as follows:

- Discuss issues and techniques that arise in doing global fits
- Emphasize topics not covered in the previous lectures
- Avoid duplication where possible
- Give updates on certain topics where new work has been done

Outline - Lecture I

1. Introduction
2. PDFs - What are they and how are they used?
3. Evolution equations and scale dependence
4. Global Fits - why do them and how are they done?
5. Overview of observables
6. Sources of uncertainty in PDFs
7. Conventions, choices, and other items affecting global fits

Outline - Lecture II

1. Treatment of Errors
   - Goodness of fit
   - Statistical errors
   - Systematic errors
   - What is *not* included
2. Overview of PDF results
3. Estimating PDF Errors
4. Outlook

Parton Distribution Functions – PDFs

- Contain nonperturbative information about the longitudinal momentum fraction distributions of quarks and gluons
- Provide a means for connecting hadron initiated processes with parton initiated subprocesses
- Universal, so many processes can be calculated with the same set of PDFs
- Allow separation of long-distance and short-distance parts of the scattering process – the short-distance parts may then be treated using perturbation theory
- PDFs provide important information as to the underlying structure of hadrons

How are PDFs Used?

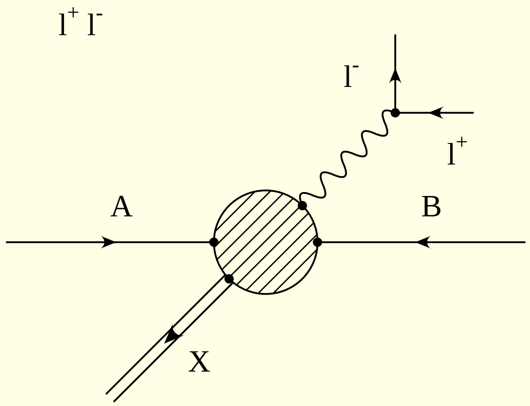
Consider a generic hard-scattering process:

$$\sigma(AB \to X) = \sum_{a,b} \int dx_a \, dx_b \, G_{a/A}(x_a, M^2) \, G_{b/B}(x_b, M^2) \, \hat{\sigma}^{ab \to X}(x_a, x_b, M^2, ...)$$

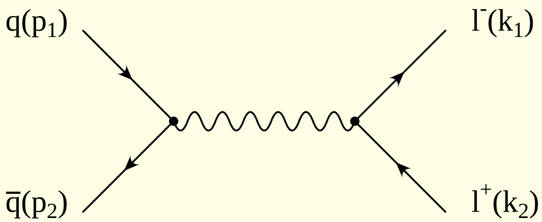
- $G_{a/A}(x_a, M^2) dx_a$ is the probability of finding a parton a in a hadron A with a momentum fraction x between x_a and $x_a + dx_a$.
- M denotes the factorization scale which serves to separate the long- and short-distance parts of the scattering process
- Partonic subprocess $\hat{\sigma}$ is convoluted with the relevant set of PDFs
- $\hat{\sigma}$ depends on the momentum fractions and the factorization scale, as well as other kinematic variables and the renormalization scale

Where does the factorization scale come from?

Consider lepton pair production: $AB \to l^+ l^- + X$

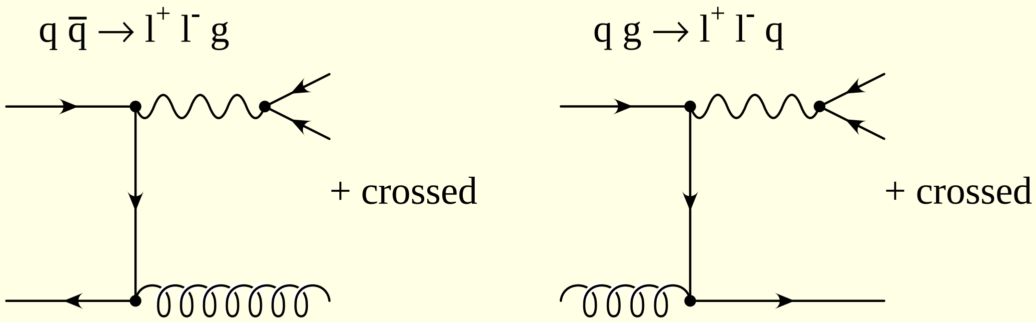


Lowest order (LO or Born term) subprocess is $q\bar{q} \to l^+ l^-$



At this level there is no factorization scale dependence in the hard-scattering subprocess. The $2 \to 2$ subprocess depends only on the partonic center-of-mass energy and the lepton scattering angle.

Next-to-Leading-Order terms (NLO)



- Lepton pair recoils against a quark or gluon
- Integration over the transverse momentum of the recoiling parton generates a logarithmic corrections originating from soft and collinear divergences - see the lectures on NLO calculations for details
- Soft divergences cancel against contributions coming from virtual corrections
- Collinear divergences are factorized and absorbed into the PDFs

- Typically see terms like $\log(Q^2/\Lambda^2) = \log(Q^2/M^2) + \log(M^2/\Lambda^2)$
- Q represents the dilepton invariant mass and Λ is the usual QCD scale parameter
- $\log(Q^2/M^2)$ represents a short-distance hard-scattering correction and is retained in the subprocess $\hat{\sigma}$
- $\log(M^2/\Lambda^2)$ terms are absorbed into the PDFs
- The factorization scale M now appears in both the hard-scattering subprocess and the PDFs
- Choosing $M = Q$ removes the logarithmic corrections from the hard scattering subprocess
- The PDFs now depend on the scale Q
- For additional details see the sample calculation in Lecture I of my Summer School 2000 presentation on the CTEQ web site (www.cteq.org).

Factorization Scale dependence of PDFs

- Running coupling decreases logarithmically: $\alpha_s(M^2) \sim 1/\log(M^2/\Lambda^2)$
- Collinear logs have been absorbed into the PDFs - one power of α_s for each collinear log
- Lose the advantage of asymptotic freedom since

$$\alpha_s(M^2)\log(M^2/\Lambda^2) \sim \text{constant}$$

- Must resum these terms to all orders
- Essential tool is the set of Altarelli-Parisi (DGLAP) evolution equations

DGLAP Equations

$$Q^2 \frac{dG_q(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} \left[P_{qq}(y) G_q(\frac{x}{y}, Q^2) + P_{qg}(y) G_g(\frac{x}{y}, Q^2) \right]$$

$$Q^2 \frac{dG_g(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} \left[\sum_q P_{gq}(y) G_q(\frac{x}{y}, Q^2) + P_{gg}(y) G_g(\frac{x}{y}, Q^2) \right]$$

- Coupled set of equations whose solutions show how the PDFs change with variations in the scale Q
- In a typical application one assumes a set of boundary conditions (parametrized PDFs) at some scale Q_0 and then the PDFs at some other scale Q are obtained as solutions.
- Splitting functions have perturbative expansions

$$P_{ij}(y) = P^0_{ij}(y) + \frac{\alpha_s}{2\pi} P^1_{ij}(y) + \cdots$$

The P_{ij}s describe the splittings $q \to qg, g \to q\bar{q},$ and $g \to gg$

- Use of $P_{ij}^0(y)$ yields leading-order (LO) PDFs – generally used in fits with LO hard scattering expressions and one-loop α_s
- Inclusion of $P_{ij}^1(y)$ yields next-to-leading-order (NLO) PDFs – generally used with NLO hard scattering expressions and two-loop α_s
- Series is known through second order – $P_{ij}^2(y)$ – would yield NNLO PDFs to be used with NNLO hard scattering expressions and three-loop α_s

Global Fits – What are they?

Problem: We need a set of evolved PDFs in order to be able to calculate a particular hard-scattering process

Solution: Generate a set of PDF solutions using a parametrized functional form for the input PDFs. Repeatedly vary the parameters and evolve the PDFs again in order to obtain an optimal fit to a set of data for various hard-scattering processes

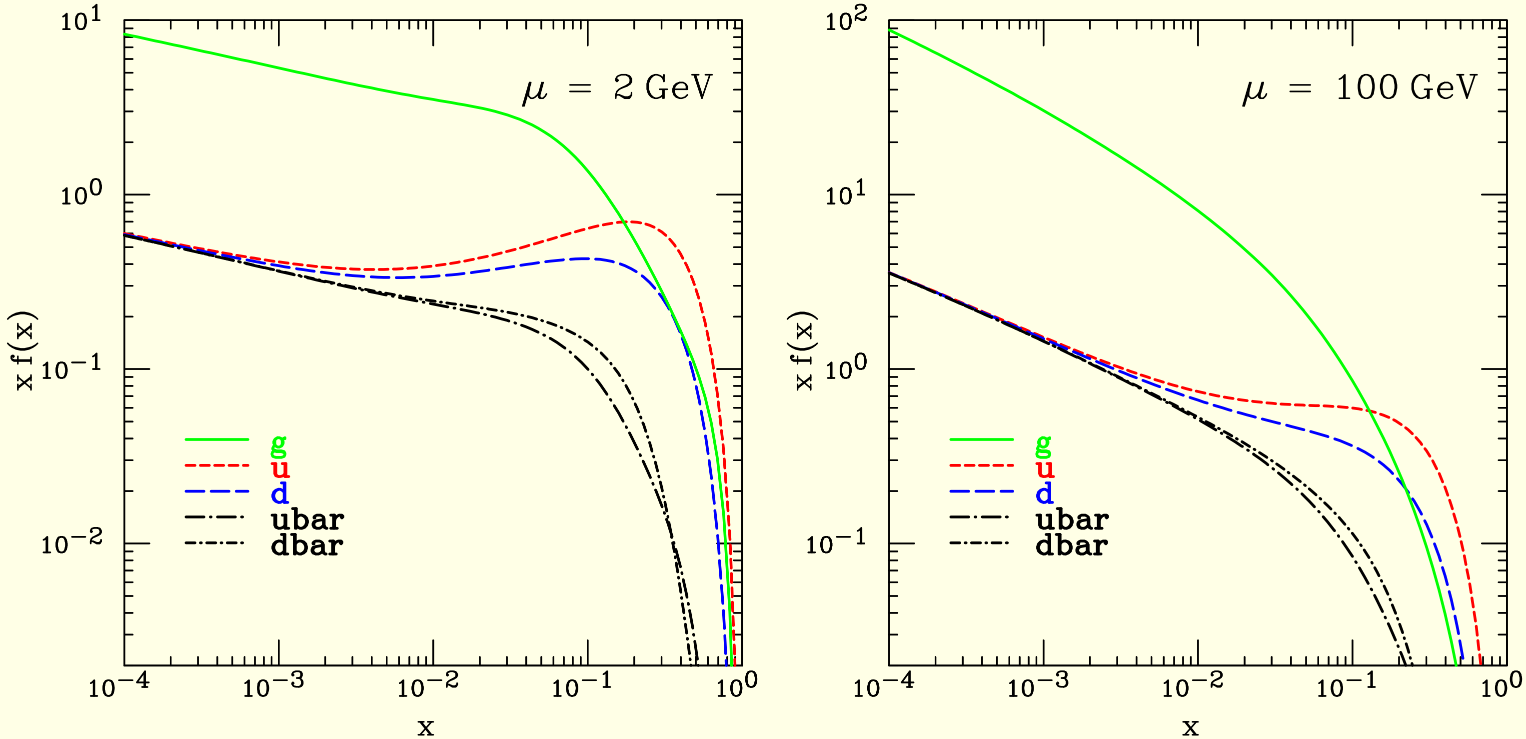
Key points:

- Parametrized functional form for input PDFs at the scale Q_0
- Choice of data sets to use and the kinematic cuts to place on them
- Truncation of the perturbation series for the hard-scattering calculations and the PDF evolution (LO, NLO, NNLO)
- Definition of "optimal fit"
- Treatment of errors

Useful PDF properties

For the specific case of the proton we know that

- The gluon distribution dominates at low values of x and falls steeply as x increases
- The antiquarks and quarks are comparable at low values of x and the antiquarks fall off in x even faster than the gluons
- the u and d PDFs dominate at large values of x with $u > d$



The pattern is easily understood by studying the evolution equations.

- The u and d distributions dominate at large x and radiate gluons as they interact in the hard-scattering process
- This causes the quark distributions to get steeper (they give up some of their momentum fraction) and the gluon distribution to get larger
- Gluons can also radiate gluons so the gluon distribution tends to also get steeper and builds up at low values of x
- Gluons can also create quark-antiquark pairs so the antiquarks increase at low values of x and have a steeper distribution than the gluons
- Keep these ideas in mind as we look for ways to separate these distributions.

Observables

Each observable involves a characteristic linear combination (or product) of PDFs. Thus, different observables can be used to constrain specific PDFs.

- Representative global fits today use data of the following types
  - Deep inelastic scattering ($l^{\pm}p$, $l^{-}d$, νN, $\overline{\nu}N$)
  - Neutrino DIS dimuon production
  - Vector boson production ($W^{\pm}$, Z^0, γ, lepton pair production)
  - hadronic jet production
- Will look at representative data types in order to design a strategy for constraining individual PDFs using the parton model as a guide
- For purposes of illustration, consider just the LO expressions for the different observables.
- Basic pattern is not altered by NLO corrections

Deep Inelastic Scattering

Lowest order $l^{\pm}p$ -

$$F_2(x,Q^2) = x\sum_i e_i^2\left[q_i(x,Q^2)+\overline{q}_i(x,Q^2)\right]$$

- Each flavor weighted by its squared charge
- Gluon doesn't enter in lowest order
- Quarks and antiquarks enter together

Move on to neutrino interactions - measure both F_2 and xF_3

$$\begin{aligned} F_2(x,Q^2) &= x\sum_i\left[q_i(x,Q^2)+\overline{q}_i(x,Q^2)\right] \\ xF_3(x,Q^2) &= x\sum_i\left[q_i(x,Q^2)-\overline{q}_i(x,Q^2)\right] \end{aligned}$$

- Additional structure function allows the separation quarks and antiquarks, but not a complete flavor separation

Flavor Separation

- In principle, these DIS measurements would be enough to separate quarks and antiquarks and also to separate charge 2/3 and charge -1/3 quarks since the $l^{\pm}$ and ν structure functions have different weighting factors
- But, the ν observables are usually obtained using nuclear targets so there is the added question of nuclear corrections - more later.
- Also have charged lepton cross sections on deuterium
- Smaller nuclear corrections than for heavier targets - still important at large values of x, however.

- After correcting the neutrino and charged lepton cross sections to effective isoscalar targets (N=$(p+n)/2$) and ignoring strange and charm contributions, one has

$$F_2^{l^\pm N} \approx \frac{5}{18}\left[u + d + \overline{u} + \overline{d}\right]$$

$$F_2^{\nu N} \approx \left[u + d + \overline{u} + \overline{d}\right]$$

- Hence $F_2^{l^\pm N} \approx \frac{5}{18} F_2^{\nu N}$ so that similar information can, in principle, be obtained from either one.

But what about the gluon?

Constraining the gluon in DIS

- The gluon does not contribute in lowest order to the DIS structure functions
- It does enter in next-to-leading order to all the structure functions
- Significant contribution to the longitudinal structure function F_L starting at order α_s, but the existing data have large errors
- Also enters through mixing in the evolution equations so the gluon contributes to the change of the structure functions as Q^2 increases

Gluon PDF and Scaling Violations

- Keep just the dominant gluon term in the quark DGLAP equation for low values of x
- Form the combination of quarks which contribute to F_2

$$Q^2 \frac{dF_2}{dQ^2} \approx \frac{\alpha_s}{2\pi} \sum e_i^2 \int_x^1 \frac{dy}{y} P_{qg}(y) G(\frac{x}{y}, Q^2)$$

- The Q^2 dependence at small-x is driven directly by the gluon PDF

Lepton Pair Production

- For pp or pd reactions lepton pair production involves the product of quark and antiquark PDFs

$$\frac{d\sigma}{dQ^2dx_f} \propto \sum_i e_i^2 \left[q_i(x_a, Q^2)\overline{q}_i(x_b, Q^2) + a \leftrightarrow b\right]$$

- x_a and x_b are given by $x_{a,b} = \frac{\pm x_F + \sqrt{x_F^2 + 4Q^2/s}}{2}$
- For large x_F one has $x_a >> x_b$ and

$$\begin{aligned} \sigma^{pp} &\propto 4u(x_a)\overline{u}(x_b) + d(x_a)\overline{d}(x_b) \\ \sigma^{pd} &\propto [4u(x_a) + d(x_a)][\overline{u}(x_b) + \overline{d}(x_b)] \end{aligned}$$

- Data sets for these processes can help determine the ratio $\overline{d}/\overline{u}$.

$W^{\pm}$ Lepton Asymmetry

- The dominant contributions to W production at the TeVatron come from $u\overline{d}$ and $\overline{u}d$ collisions.
- But $G_{\overline{u}/\overline{p}}(x) = G_{u/p}$ and similarly for the d quark.
- Hence, at the TeVatron one is sensitive to the product ud
- Define the W asymmetry in rapidity y as

$$A_W(y) = \frac{d\sigma^+/dy - d\sigma^-/dy}{d\sigma^+/dy + d\sigma^-/dy}$$

- For $p\overline{p}$ collisions one has

$$A_W(y) \approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)}$$

- Here $x_{a,b} = x_0 e^{\pm y}$ with $x_0 = M_W/\sqrt{s}$.

- Let $R_{du} = d/u$ so that

$$A_W(y) = \frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)}$$

- For small y one has $R_{du}(x_a) \approx R_{du}(x_b) \approx R_{du}(x_0)$.
- Using a Taylor Series expansion one gets

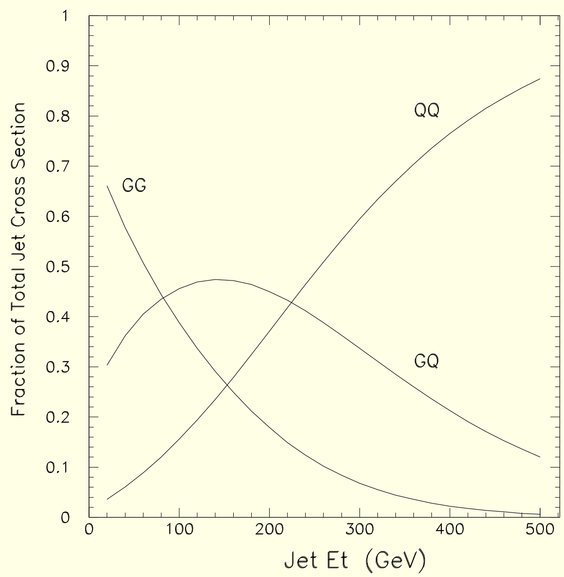
$$A_W(y) \approx -x_0 y \frac{1}{R_{du}(x_0)} \frac{dR_{du}}{dx}(x_0)$$

- The W asymmetry thus yields information on the slope of the d/u ratio.
- The same conclusion holds for the lepton asymmetry from the W decay, but the effect is washed out somewhat by the decay.

Hadronic Production of Jets

So far we have not obtained much information about the gluon distribution. Need a process where the gluon contributes in lowest order.

- Direct photon production is one candidate - more will be said later about this.
- Hadronic jet production includes, in lowest order, $qq \to qq$, $qg \to qg$, and $gg \to gg$
- At high $x_T = 2p_T/\sqrt{s}$ one might expect the quark distributions to dominate since the relevant values of x are of order x_T.



- The qq subprocesses do dominate the high-E_T region.
- But, there is enough contribution from the gluon that high-E_T jet data can be used to constrain the large-x gluon behavior.
- Combined with the low-x data and the momentum sum rule (to be discussed later) one has strong constraints on the gluon distribution.

Global Fits

Ok - so you think you are ready to do some global fits...

- $\surd$ Collected data for a representative set of processes
- $\surd$ Obtained an evolution program for the PDFs
- $\surd$ Written or obtained a set of programs to evaluate the various observables
- $\surd$ Interfaced a fitting package with the observable and evolution routines
- $\surd$ Ready to go - right?

Oh, but wait. Not so fast . . .

There are just a few details left to address

- Parametrization and choice of parameters to vary
- Order of perturbation theory (LO, NLO, NNLO, ...)
- Scheme dependence (DIS, $\overline{\mathrm{MS}}$, ...)
- Choices for scales in the hard scattering processes
- Target mass and higher twist effects
- Treatment of heavy quarks
- Effects due to choosing or deleting a given data set
- Choice of kinematic cuts
- Treatment of errors
- Error estimates on the PDFs

Uniqueness of PDFs

If we all fit the same data and we all use the same theory, why are there differences between PDF sets?

- Different constraints are imposed on the PDFs ($\overline{u} = \overline{d}\,?$, $\overline{s} = s\,?$, parametrization of low-x and high-x behaviors, etc.)
- Different choices of data sets will provide different constraints on the PDFs
- Different levels of approximation are often used - LO, NLO, NNLO, inclusion of target mass effects, neglect of quark mass effects, etc.
- Not all questions have unambiguous answers - choice of scale for each process, kinematic range considered (Q^2 and W^2 cuts in DIS for example)

All of these can affect the PDFs that result from the global fit. The user must be aware of the choices made for a specific set of PDFs.

Parametrizations

Initial attempts were all of the form $G(x) \sim x^{\alpha}(1-x)^{\beta}$

- Estimate β from quark counting rules wherein $\beta = 2n_s - 1$ with n_s being the minimum number of spectator quarks
- Valence quarks in a proton (qqq): $n_s = 2, \beta = 3$
- Gluon in a proton ($qqqg$): $n_s = 3, \beta = 5$
- Antiquarks in a proton ($qqqq\bar{q}$): $n_s = 4, \beta = 7$
- Estimate α from Regge arguments - gluon and antiquarks have $\alpha \approx -1$ while valence quarks have $\alpha \approx -1/2$
- Overall normalization fixed by sum rules

Sum Rules

- Net number of quarks in a proton

$$\int_0^1 [u(x) - \overline{u}(x)]\, dx = 2; \quad \int_0^1 \left[d(x) - \overline{d}(x)\right] dx = 1; \quad \int_0^1 [s(x) - \overline{s}(x)]\, dx = 0; \text{ etc}$$

- Momentum sum rule

$$\int_0^1 x \left[\sum_i (q_i(x) + \overline{q}_i(x)) + g(x)\right] dx = 1$$

- The number sum rules help fix the normalizations of the quark distributions
- The momentum sum rule helps fix the gluon normalization
- The momentum sum rule ties the low-x and high-x gluon behaviors together
- The s and $\overline{s}$ distributions need not be identically equal – they just have to satisfy the integral given above.

Parametrizations (continued)

- Simple form is often not sufficient
- Usually multiply the simpler forms by a polynomial in x or some more complicated function
- CTEQ uses the following for the quarks and gluon distributions

$$x^{(a_1-1)}(1-x)^{a_2}e^{a_3x}\left[1+e^{a_4}x\right]^{a_5}$$

- Also $\overline{d}/\overline{u} = e^{a_1}x^{(a_2-1)}(1-x)^{a_3} + (1+a_4x)(1-x)^{a_5}$
- Choice of functional form or number of parameters can bias the results
- For example, $\overline{d}/\overline{u}$ above goes to 1 as $x \to 0$. Is that right? Change $(1+a_4x)$ to (a_6+a_4x) and see. (It doesn't seem to make a difference with the data used.)

Parametrizations (continued)

- Increase the number of parameters and the flexibility of the parametrization until the data are well described
- Adding more parameters past that point simply results in ambiguities, false minima, unconstrained parameters, etc.
- May encounter "flat directions" in parameter space - χ^2 may be insensitive to a parameter or a combination of parameters.
- Wish to avoid this situation as it causes problems with the convergence of the fitting program
- May have to make some arbitrary decisions on parameter values that are not well constrained by the data
- A smaller numbers of parameters is not always better - it is the description of the data that counts.

Order of Perturbation Theory

- Lowest order in α_s (LO) - easy to do, but
  - Hard scattering subprocesses do not depend on the factorization scale
  - May be missing large higher order corrections
- Next-to-leading-order (NLO) - more complicated, but
  - Less dependent on scale choices since the PDFs and hard scattering subprocesses both contain scale dependences which (partially) cancel
  - Some higher order corrections are now included
- Next-to-next-to-leading-order (NNLO) - better, but
  - Splitting functions are known so NNLO evolution can be done
  - Some hard scattering subprocesses are known to NNLO (DIS, lepton pair production) but not high-E_T jets (yet)
- NLO remains the state-of-the-art, but full NNLO analyses are coming

Scheme Dependence

- When doing perturbative calculations one needs to specify a scheme or convention for subtracting the divergent terms.
- Basically, the scheme specifies how much of the finite corrections to subtract along with the divergent pieces
- Most widely used scheme is the modified minimal subtraction scheme or $\overline{\text{MS}}$. Used with dimensional regularization - subtract the pole terms and accompanying $\log 4\pi$ and Euler constant terms.
- Occasionally one will encounter the DIS scheme - the full $\mathcal{O}(\alpha_s)$ correction for F_2 in DIS is absorbed into the quark PDFs.
- It is important to know which scheme was used for a set of PDFs since that determines what hard scattering cross sections the PDFs were convoluted with.
- CTEQ typically provides CTEQ#L, CTEQ#D, and CTEQ#M (Leading Order, DIS, and $\overline{\text{MS}}$ distributions)

Factorization and renormalization scale dependences

- Processes used in global fits are characterized by a single large scale
  - DIS - Q^2
  - Lepton pair production - M^2
  - Vector boson production - M_V^2
  - Jet production - p_T
- Choose factorization and renormalization scales to be of the same order as the characteristic scale
  - Removes large logs from the hard scattering cross section
  - Resums large logs in the running coupling and the PDFs

How does this work?

Example - high-p_T jets

Consider just the non-singlet quark-quark contribution

$$E\frac{d^3\sigma}{dp^3} \equiv \sigma = a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M)$$

where $a(\mu) = \alpha_s(\mu)/2\pi$ and the lowest order parton-parton scattering cross section is denoted by $\hat{\sigma}_B$.

The renormalization and factorization scales are denoted by μ and M, respectively.

The symbol $\otimes$ denotes a convolution defined as

$$f \otimes g = \int_x^1 \frac{dy}{y}\, f(\frac{x}{y})\, g(y).$$

When one calculates the $\mathcal{O}(\alpha_s^3)$ contributions to the inclusive cross section, the result can be written as

$$
\begin{aligned}
\sigma &= a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
&+ 2a^3(\mu)\,b\ln(\mu^2/p_T^2)\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
&+ 2a^3(\mu)\,\ln(p_T^2/M^2)P_{qq} \otimes \hat{\sigma}_B \otimes q(M) \otimes q(M) \\
&+ a^3(\mu)\,K \otimes q(M) \otimes q(M).
\end{aligned}
$$

The explicit μ and M dependences have been separated from the remainder of the α_s^3 corrections which are denoted by K.

Here P_{qq} is the quark splitting function from the DGLAP equations and b is the one-loop contribution to the QCD beta function.

One sees explicit logs which partially cancel the running coupling and PDF scale dependences

If μ and M are chosen to be p_T then the p_T-dependent logs are resummed in the running coupling and the PDFs.

The running coupling satisfies

$$\mu^2 \frac{\partial a(\mu)}{\partial \mu^2} = \beta(a(\mu))$$

where $\beta = -ba^2 + \ldots$ with $b = \frac{33-2f}{6}$

and where f denotes the number of flavors.

The scale dependence of the quark distributions is given by the nonsinglet DGLAP equation

$$M^2 \frac{\partial q(x, M)}{\partial M^2} = a(M) P_{qq} \otimes q(M).$$

Homework Problem

Use the previous equations to show that for the case of $\mu = M$ one has

$$\mu^2 \frac{\partial \sigma}{\partial \mu^2} = 0 + \mathcal{O}(a^4).$$

Actually, one can also show that this will be true for the μ and M dependences separately.

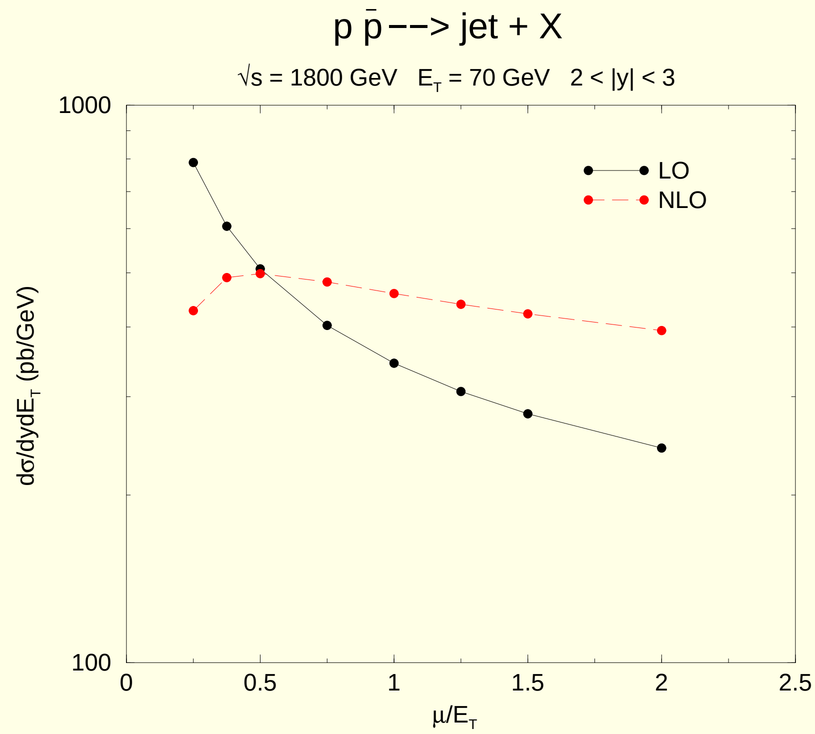
Choosing the scales the exactly satisfy these equations results in the PMS (Principle of Minimal Sensitivity) algorithm for choosing the scales.

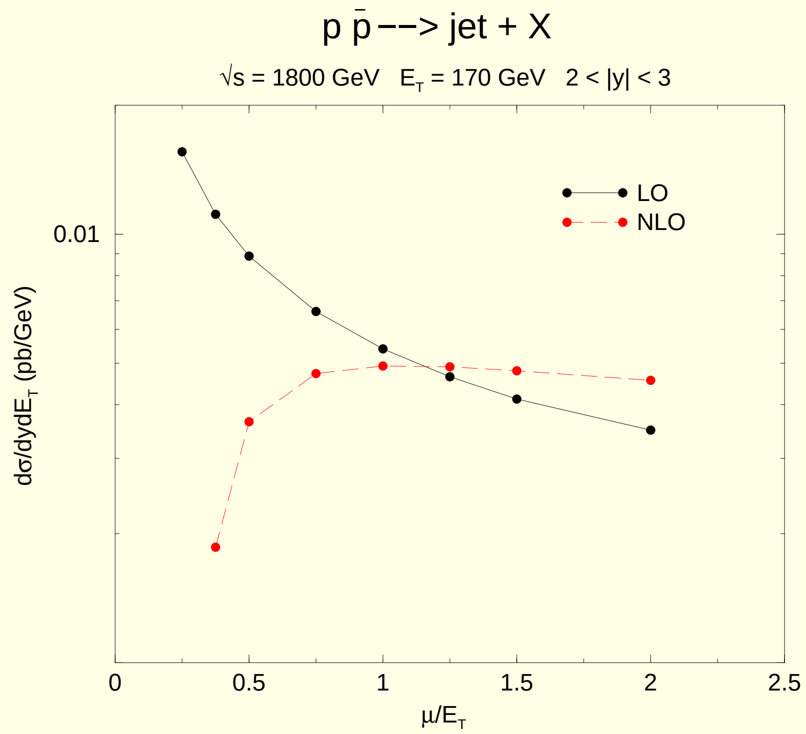
Systematics of NLO cross sections

$$\begin{aligned}
\sigma &= a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
&+ 2a^3(\mu)\,b\ln(\mu^2/p_T^2)\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
&+ 2a^3(\mu)\,\ln(p_T^2/M^2)P_{qq} \otimes \hat{\sigma}_B \otimes q(M) \otimes q(M) \\
&+ a^3(\mu)\,K \otimes q(M) \otimes q(M).
\end{aligned}$$

The first line is the Born or LO contribution. At large values of P_T the quark PDFs and the running coupling decrease with increasing scales. The same behavior occurs for the last line.

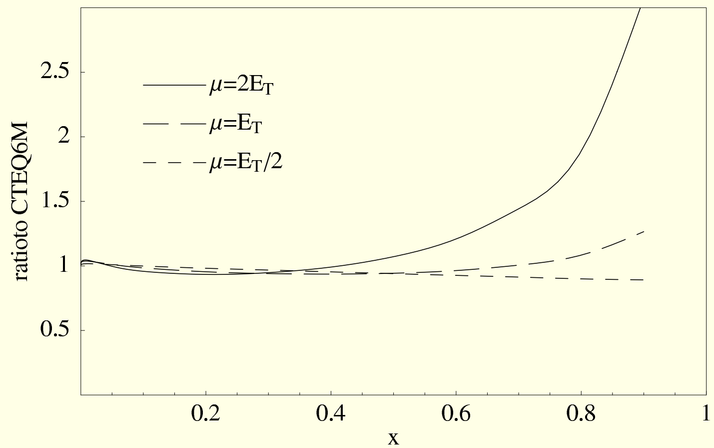
Lines 2 and 3 give negative contributions when the scales are smaller than p_T and turn positive when they are larger than p_T.





- Each figure shows the cross section at a fixed value of E_T as the scale choice is varied ($M = \mu$ is used here.)
- As expected, the LO curve is a monotonically decreasing function of the scale choice.
- The NLO curves are much flatter showing a maximum near $\mu \approx E_T$ with a decrease for both larger or smaller choices.

If the cross section depends on the choice of scale, then as the scale is varied the PDFs will have to change in order to be able to fit the data.



- CTEQ6 used $\mu = M = E_T/2$
- Redoing the fit with $\mu = M = E_T$ or $2E_T$ causes the high-x gluon to be larger since the high-E_T partonic cross sections have decreased as shown previously.
- The scale dependence results in a shift of the PDFs and, hence, makes a contribution to the PDF uncertainty.
- Note: this contribution is not included in the standard PDF errors.

Target Mass and Higher Twist Effects

- Target mass effects take into account the finite mass of the target nucleon
- Main effect is to modify the scaling variable x to the Nachtmann variable ξ

$$x \rightarrow \xi = x \left[\frac{2}{1 + \sqrt{1 + 4x^2 m^2 / Q^2}} \right]$$

  where m is the target mass.

- DIS structure functions are also modified

$$\begin{aligned} F_1(x, Q^2) &= F_1^{m=0}(\xi, Q^2) \\ F_2(x, Q^2) &= \frac{1}{1 + 4x^2 m^2 / Q^2} \frac{x}{\xi} F_2^{m=0}(\xi, Q^2) \end{aligned}$$

- See the talk by Jian-Wei Qiu at the 2005 Jefferson Lab meeting on the CTEQ web site

Target mass and higher twist (continued)

Nachtmann variable

$$x \to \xi = x \left[\frac{2}{1 + \sqrt{1 + 4x^2 m^2 / Q^2}} \right]$$

- Note that $\xi \to x$ as Q^2 increases
- The structure functions smoothly go over to their $m = 0$ forms
- Effects are largest in the region near $x \approx 1$

Higher Twist Terms

- Terminology dates back to the Operator Product Expansion
- Refers to an expansion in inverse powers of Q^2
- Leading twist term is the scaling parton model result (with logarithmic QCD corrections)

Higher twist terms (continued)

- Higher twist operators would contribute terms to DIS structure functions of the form

$$f(x)/Q^2,\ g(x)/Q^4,\ \ldots$$

- Generally related to correlations between partons in the parent nucleon
- Some analyses include parametrized functions such as $f(x)$ above
- No general consensus even as to the sign of such terms since there is no unique signature in the data
- Easily confused with target mass effects (which actually are an example of kinematic higher twist terms)

Often limit the contribution of target mass and higher twist terms by placing cuts on Q^2 and $W^2 = m^2 + Q^2(1-x)/x$

Typically $Q^2 > 5$ GeV^2 and $W > 3.5$ GeV which results in $x < 0.76$ which matches the range of x covered by many high energy DIS experiments

Quark Masses

Zero Mass-Variable Flavor Number Scheme (ZM-VFNS)

- Typically start the PDF evolution at the charm threshold $Q = m_c = 1.3$ GeV
- Set c and b distributions equal to zero (no intrinsic heavy flavors)
- Heavy $q\bar{q}$ pairs created by $g \to q\bar{q}$
- Treat all quarks as massless
- Adjust the running coupling as each flavor threshold is crossed since the QCD β function depends on the number of active flavors
- Start b evolution at $Q = m_b$
- In this approach the only mass effects are due to the imposition of the flavor thresholds and the changing of the β function which controls the running of α_s.

Advantages of the ZM-VFNS

- Easy to implement within the usual zero mass scheme used for the light quarks
- Sums large logs of Q^2/m_Q^2 via the DGLAP equations
- Asymptotically correct when $Q^2 >> m_Q^2$

Disadvantages

- Does not treat the heavy quark threshold correctly where

$$\hat{W}^2 = Q^2(\frac{1}{x} - 1) > 4m_Q^2$$

gives the threshold for the photon gluon fusion subprocess $\gamma^* g \to Q\overline{Q}$

- This is replaced by $Q^2 > m_Q^2$

Would like to have a scheme which is correct near threshold yet reduces to the ZM-VFNS result for large Q^2

Fixed Flavor Number Scheme (FFNS)

- Calculate the heavy quark production from the relevant subprocesses such as $\gamma^* g \to Q\overline{Q}$ keeping only light quarks in the DGLAP equations
- Only light quarks have PDFs - there would be no charm or bottom PDFs

Advantage - gets the threshold behavior correct

Disadvantage - does not resum potentially large logs of Q^2/m_Q^2

Variable Flavor Number Scheme - (VFNS)

- This combines the ZM-VFNS and FFNS schemes by interpolating between the FFNS (correct near threshold) and the ZM-VFNS (resums large logs)
- Technically more complicated than the ZM-VFNS since there must be subtraction terms in order to avoid double counting

Consider charm production in the ZM-VFNS. The charm distribution in LO gives a contribution to F_2 which is

$$F_2(x, Q^2) = 2x\, e_c^2\, c(x, Q^2)$$

where $c(x, Q^2)$ is calculated from the DGLAP equations. This charm PDF resums large logs of Q^2/m_c^2. But near threshold it is easy to see what it looks like. Since the charm PDF starts at zero at threshold, one starts with

$$Q^2 \frac{d\, c(x, Q^2)}{dQ^2} \approx \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y}\, P_{qg}(y)\, g(\frac{x}{y}, Q^2)$$

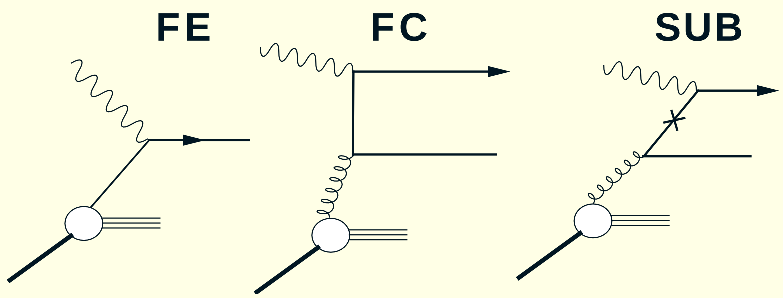
Integrating on Q^2 starting at m_c^2 gives an $\mathcal{O}(\alpha_s)$ result of

$$c(x, Q^2) \approx \frac{\alpha_s}{2\pi} \log(\frac{Q^2}{m_c^2}) \int_x^1 \frac{dy}{y} \, P_{qg}(y) \, g(\frac{x}{y}, Q^2)$$

Repeated iterations shows how the logs are resummed. Of course, as one goes higher in Q^2 the charm PDF contribution in the integrand must be added.

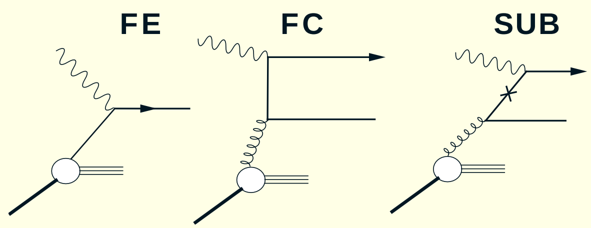
Note: one can use $g(\frac{x}{y}, Q^2)$ or $g(\frac{x}{y}, m_c^2)$. The difference is higher order in α_s.

Now, one would like to add a correction term to the LO contribution from the charm PDF. This correction term would come from the $\mathcal{O}(\alpha_s)$ mechanism for charm production $\gamma^* g \to c\bar{c}$.



- The first term is just the LO charm contribution (FE = Flavor Excitation)
- The second term is the photon-gluon fusion term (FC = Flavor Creation)
- The third term is a subtraction term which removes the double counting inherent in the first two
- The subtraction term will be proportional to the one-loop charm contribution given previously.

Heavy Quarks (continued)



- Near threshold the first and third terms cancel leaving just the photon-gluon fusion contribution.
- At large Q^2 the photon-gluon fusion result approaches that of the massless subtraction term leaving only the ZM-VFNS contribution from the first diagram.
- Can generalize this approach to include the $\gamma^* c \to cg$ contribution

An excellent review of the formalism with applications in an up-to-date global fit (CTEQ6.5M) maybe found in W.K. Tung et al., hep-ph/0611254, JHEP 0702:053, 2007.

Effects of Adding or Deleting a Given data Set

- Some sets of PDFs may differ simply because different data sets have been fit.
  - Not including the hadronic production of jets can lead to a softer gluon distribution
  - Not including lepton pair production data such as that from E-866 taken on proton and deuterium targets may result in $\overline{u} = \overline{d}$.
  - Not including neutrino dilepton production may result in $s = \overline{s}$.
- Specific data sets may have particular characteristics which cause shifts in the shapes of the fitted PDFs. So, adding them or deleting them can change the final PDF results.

The variations of the type listed above will in general *not* be reflected in the error bands calculated on the basis of the errors on the data which are included in the fit. You can't know about some effect if the relevant data aren't in the fit.

Effects of kinematic cuts

- As discussed previously, cuts in variables such as Q^2 or W^2 may be imposed in order to ensure that one excludes regions where target mass effects or higher twist effects are important.
- This removes some model dependency from the results.
- However, it is important to verify that the results are stable under reasonable changes in the cuts.
- Such studies have been done by both CTEQ and MRST.

Note: It is important to understand over which kinematic ranges the PDFs have been fit.

Extrapolation outside the fitted range in x, for example, can be very dangerous. Likewise, extrapolating to values of Q^2 below the lowest values fitted can be misleading.

Concluding Comments for Lecture I

- You have seen the basic mechanics of doing global fits, how they are done, and why they are useful
- You have seen how different types of data can be used to constrain the different PDFs
- I have touched on many of the items and choices which can affect the outcome of a global fit
- In the next lecture I will give examples of results from various global fits and discuss issues related to PDF errors