

# Status of the TECHQM 'brick problem'

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# TECHQM

[https://wiki.bnl.gov/TECHQM/index.php/Main\\_Page](https://wiki.bnl.gov/TECHQM/index.php/Main_Page)

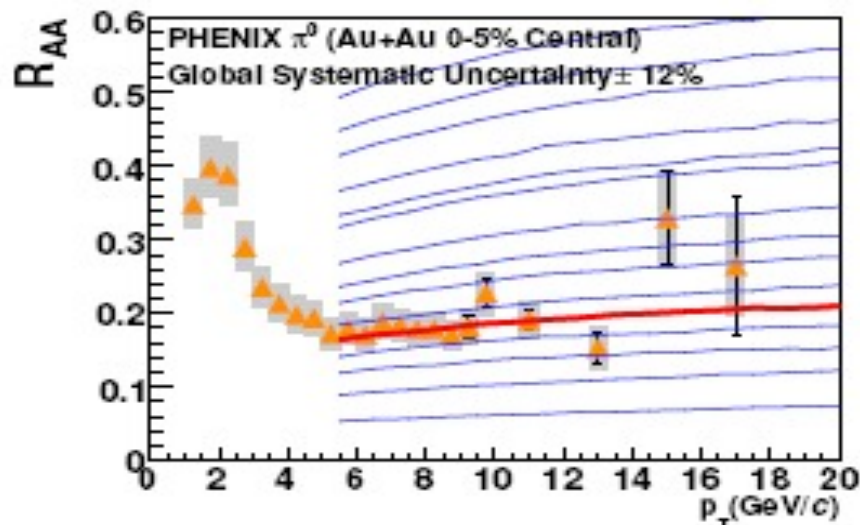
Theory-Experiment Collaboration on Hot Quark Matter

- Forum to discuss comparison between theory and experiment in areas where there is a **potential significant quantitative understanding**
- Two subgroups:
  - Parton energy loss
  - Elliptic flow/Hydro
- Workshops/meetings:
  - BNL May 2008
  - LBL Dec 2008
  - CERN July 2009
  - BNL (with CATHIE) Dec 2009

This talk is about Parton Energy loss

# Energy loss formalisms I

PHENIX, arXiv:0801.1665,  
J. Nagle WWND08



$$\text{PQM} \quad \langle \hat{q} \rangle = 13.2^{+2.1}_{-3.2} \text{ GeV}^2/\text{fm}$$

$$\text{WHDG} \quad dN_g/dy = 1400^{+200}_{-375}$$

$$\text{ZOWW} \quad \varepsilon_0 = 1.9^{+0.2}_{-0.5} \text{ GeV}/\text{fm}$$

$$\text{AMY} \quad \alpha_s = 0.280^{+0.016}_{-0.012}$$

Large difference in medium density:

GLV, AMY:  $T = 300\text{-}400 \text{ MeV}$     BDMPS:  $T \sim 1000 \text{ MeV}$

Different calculations use different geometries – not clear what dominates

# Energy loss formalisms II

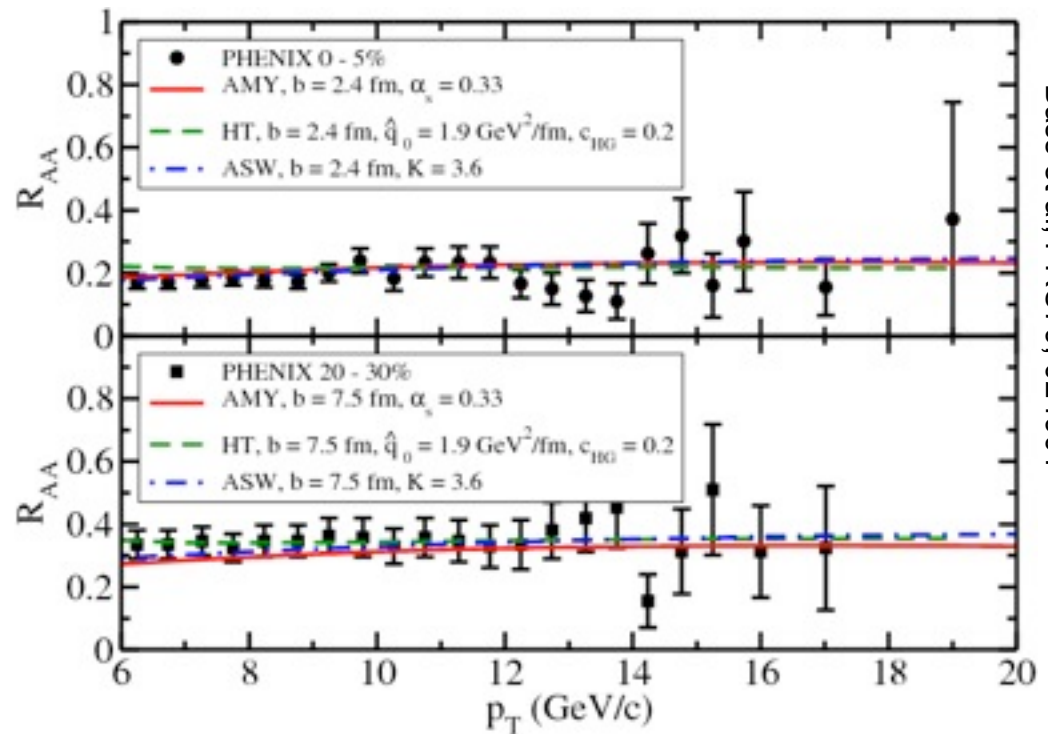
Compare 3 formalisms with  
'same' Hydro geometry:

ASW:  $\hat{q} = 10 - 20 \text{ GeV}^2/\text{fm}$

HT:  $\hat{q} = 2.3 - 4.5 \text{ GeV}^2/\text{fm}$

AMY:  $\hat{q} \approx 4 \text{ GeV}^2/\text{fm}$

AMY:  $T \sim 400 \text{ MeV}$



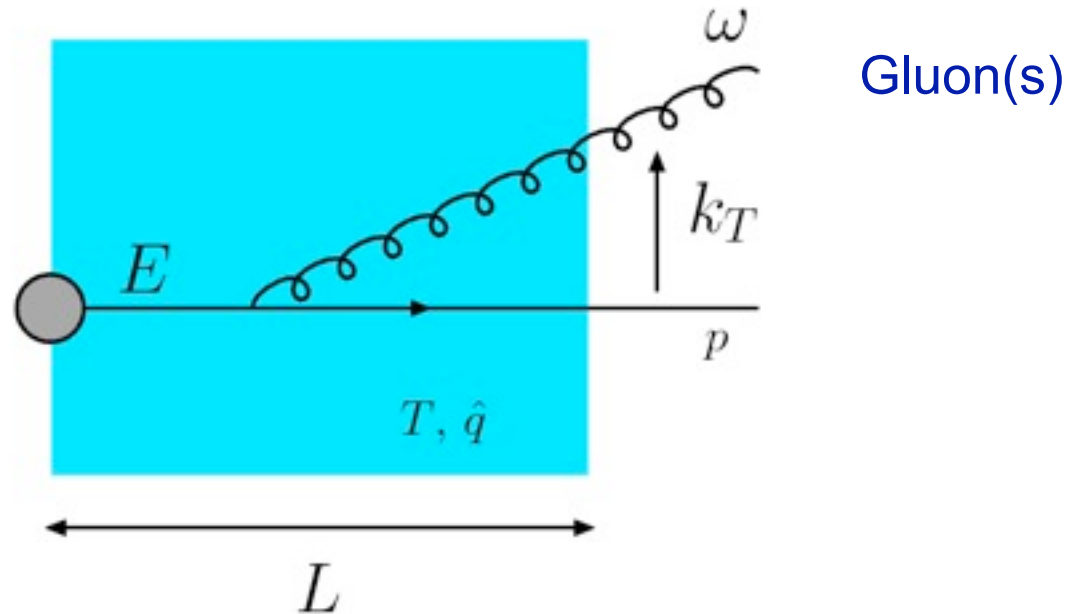
Different formalisms give different energy loss at given density, path length

Why:

Different physics implemented? Or 'technical' differences?

What are the main uncertainties?

# The Brick Problem



Compare energy-loss in a well-defined model system:

Fixed-length  $L$  (2, 5 fm)

Density  $T, \hat{q}$

Quark,  $E = 10, 20$  GeV

Plot: outgoing gluon, quark distributions

Two types of comparison:

- Same density
- Same suppression

# Four formalisms

Multiple gluon emission

- **Hard Thermal Loops (AMY)**

- Dynamical (HTL) medium
- Single gluon spectrum: BDMPS-Z like path integral
- No vacuum radiation

Fokker-Planck  
rate equations

- **Multiple soft scattering (BDMPS-Z, ASW)**

- Static scattering centers
- Gaussian approximation for momentum kicks
- Full LPM interference and vacuum radiation

Poisson ansatz  
(independent emission)

- **Opacity expansion ((D)GLV, ASW-OE)**

- Static scattering centers, Yukawa potential
- Expansion in opacity  $L/\lambda$   
( $N=1$ , interference between two centers default)
- Interference with vacuum radiation

- **Higher Twist (Guo, Wang, Majumder)**

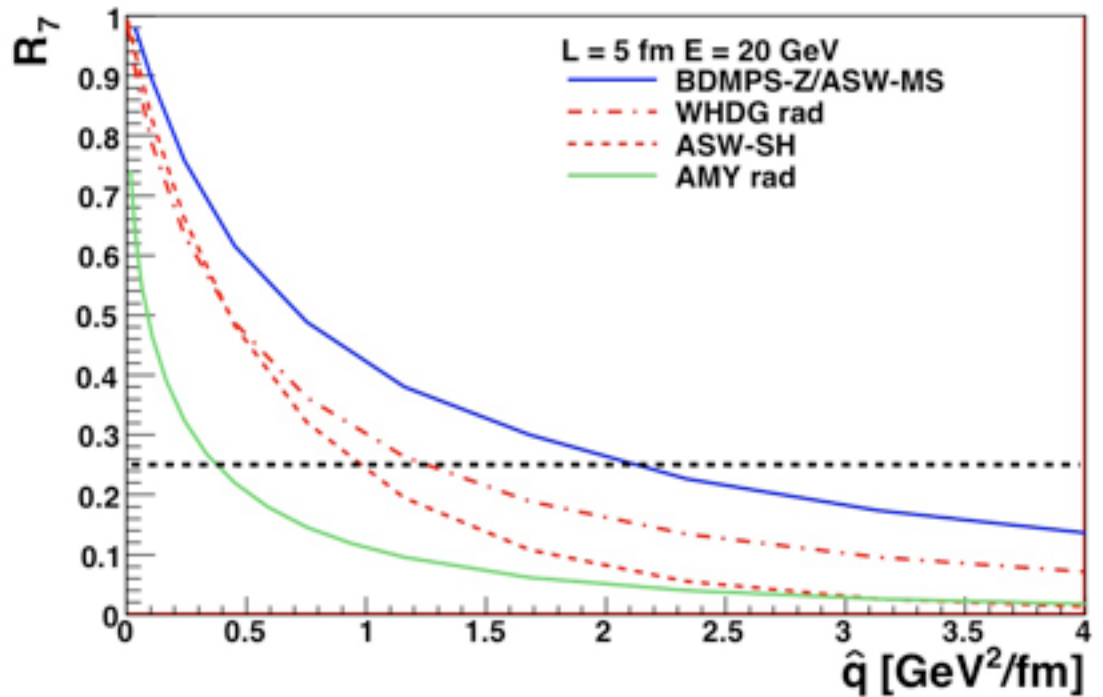
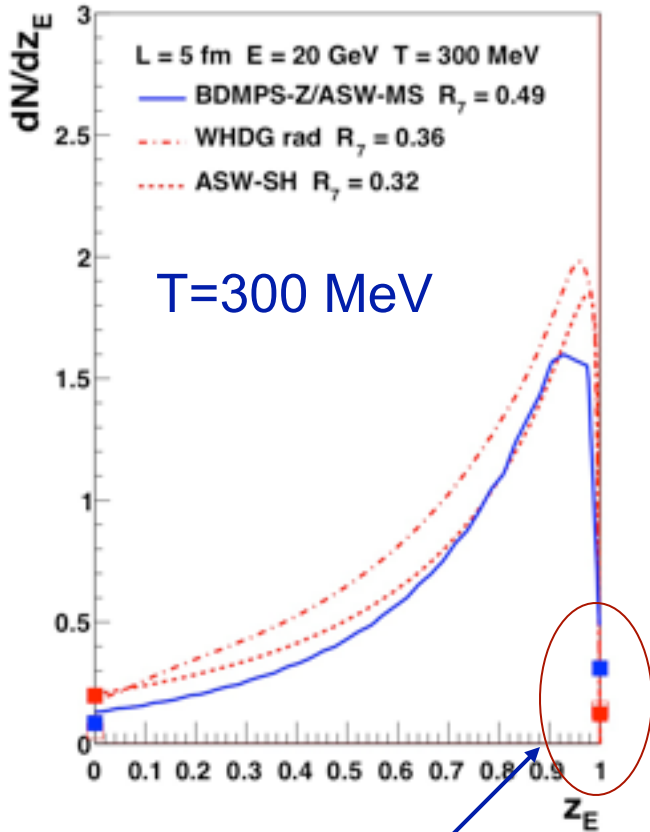
- Medium characterised by higher twist matrix elements
- Radiation kernel similar to GLV
- Vacuum radiation in DGLAP evolution

DGLAP  
evolution

# Some brick results

## Outgoing quark spectrum

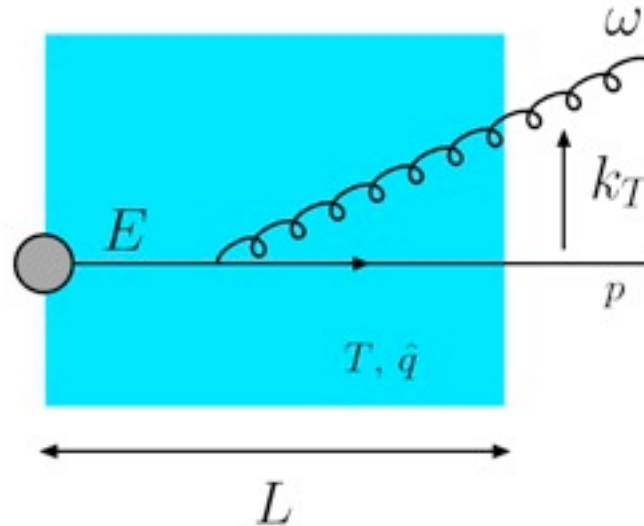
$$R_n = \int_0^1 d\varepsilon (1-\varepsilon)^{n-1} P(\varepsilon) \quad R_n \approx R_{AA} \text{ for } \frac{1}{p_T^n} \text{ spectrum}$$



Large differences in medium density  
for  $R_7 = 0.25$

→ Difference between formalisms sizable even in simple geometry

# Limitations of soft collinear approach



Calculations are done in soft collinear approximation:

$$\text{Soft: } \omega \ll E$$

$$\text{Collinear: } k_T \ll \omega$$

Need to extend results to full phase space to calculate observables (especially at RHIC)

**Soft approximation not problematic:**

For large  $E$ , most radiation is soft

Also:  $\omega > E \Rightarrow$  full absorption

**Cannot enforce collinear limit:**

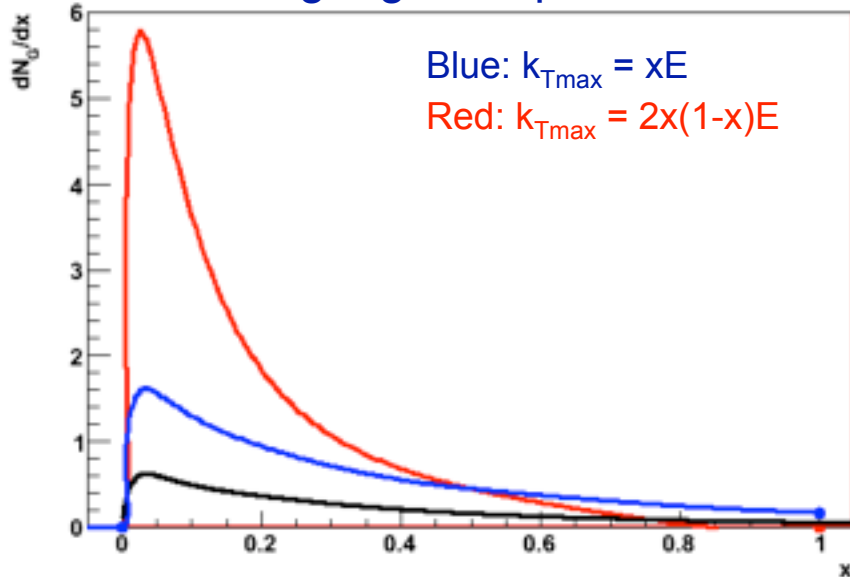
Small  $\omega$ ,  $\omega \rightarrow k_T$  always a part of phase space with large angles



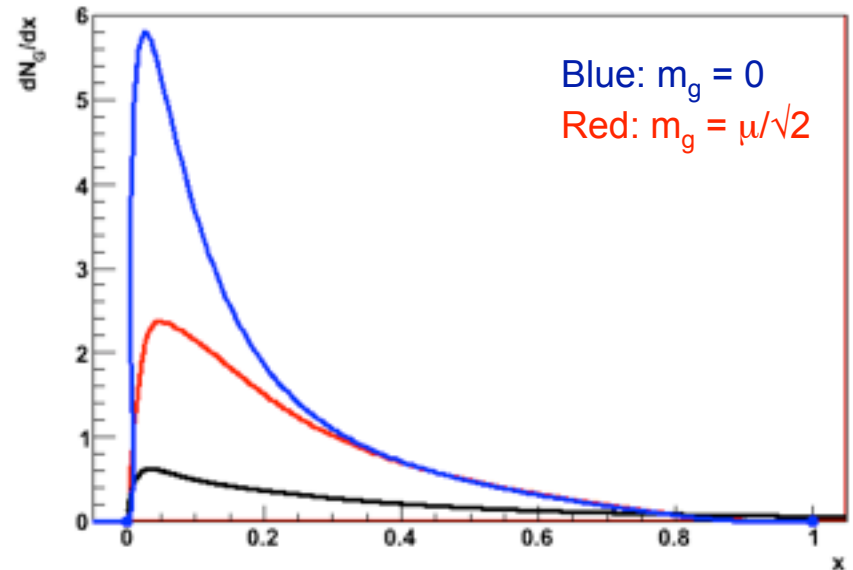
# Opacity expansions

## GLV and ASW-SH

Single-gluon spectrum



Single-gluon spectrum



Horowitz and Cole, PRC81, 024909

Different definitions of  $x$ :

ASW:  $x_E = \frac{\omega}{E}$       GLV:  $x_+ = \frac{\omega_+}{E_+}$

$$x_E = x_+ \left( 1 + \left( \frac{k_T}{x_+ E^+} \right)^2 \right)$$

$x_+ \sim x_E$  in soft collinear limit,  
 but not at large angles

Different large angle cut-offs:

$k_T < \omega = x_E E$

$k_T < \omega = 2x_+ E$

Factor  $\sim 2$  uncertainty  
 from large-angle cut-off

# Opacity expansion vs multiple soft

OE and MS related via path integral formalism

Salgado, Wiedemann, PRD68, 014008

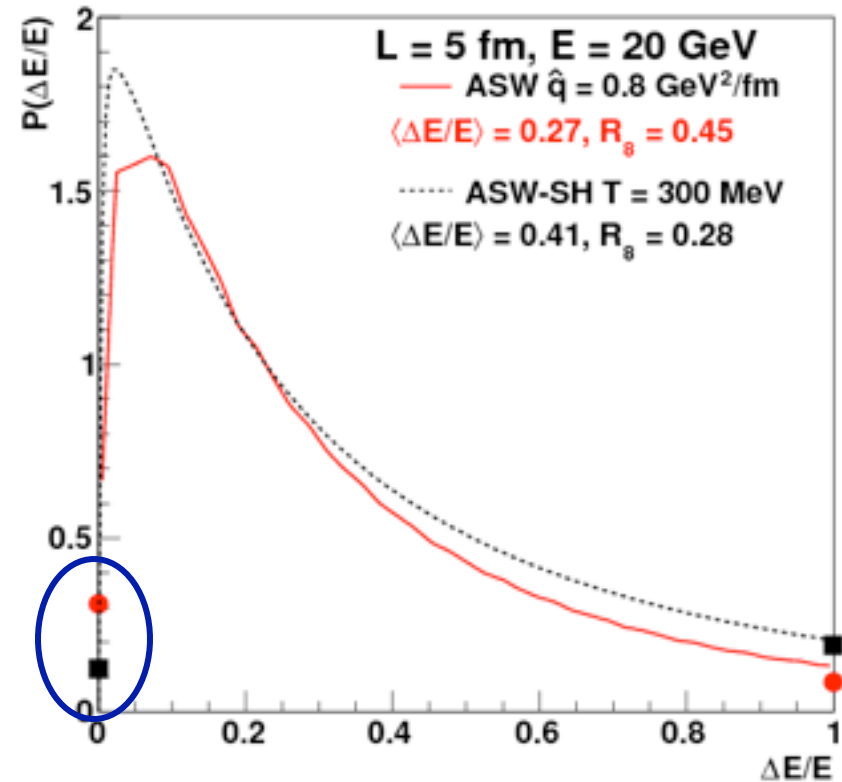
Different limits:

SH (N=1 OE): interference between neighboring scattering centers

MS: 'all orders in opacity', gaussian scattering approximation

So far, not clear which difference dominates.

Would like: OE with gaussian and/or all orders (Wicks)



Quantitative differences sizable

# AMY and BDMPS

Single-gluon kernel from AMY  
based on scattering rate:

$$\frac{q_5 d_T}{q_{\underline{L}^{ej}}} = \frac{(\gamma_{\underline{L}})_5}{I} \frac{d_5^T (d_5^T + \omega_5^D)}{d_5^T \omega_5^D}$$

BMPS-Z use harmonic oscillator:

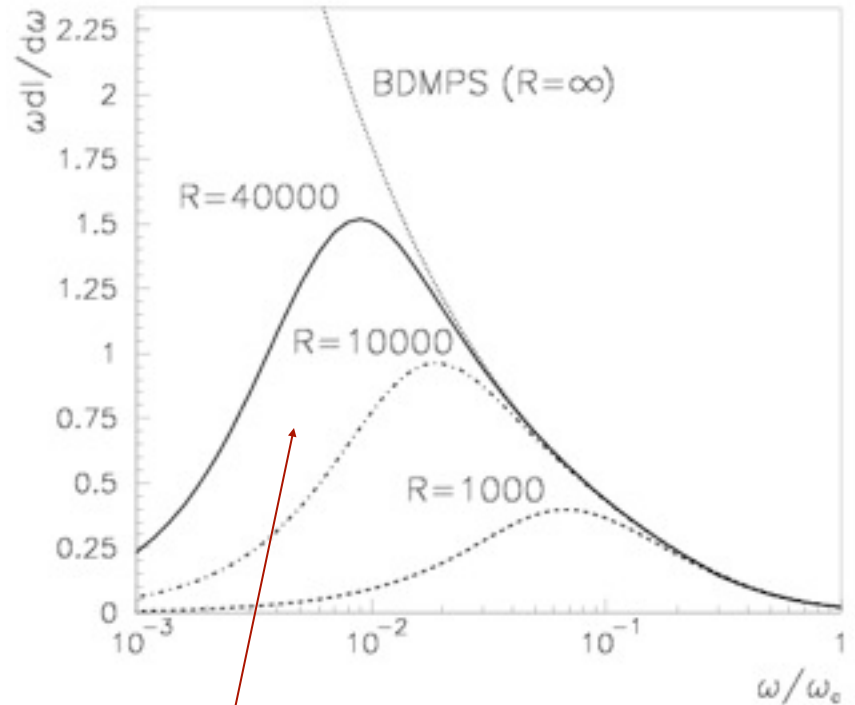
$$\underline{L}^5(\mathbf{p}, t) = \int q_5 d_T \frac{q_5 d_T}{q_{\underline{L}^{ej}}} (I - \epsilon_{ij} \mathbf{p} \cdot \mathbf{d}_T)$$

$$\bar{\Gamma}_2(\mathbf{b}, t) = \frac{1}{4} \hat{q} b^2$$

BDMPS-Z:

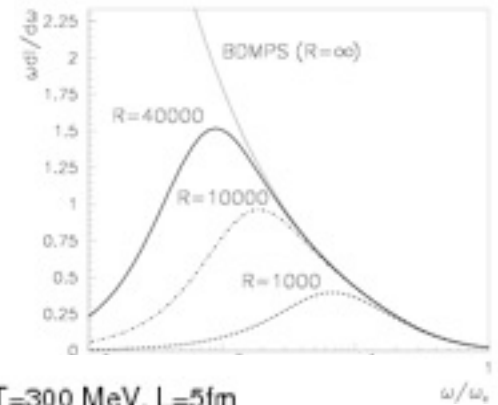
$$k \frac{dI}{dk} = \frac{\alpha x P_{s \rightarrow g}(x)}{\pi} \ln |\cos(\omega_0 L)|$$

Salgado, Wiedemann, PRD68, 014008

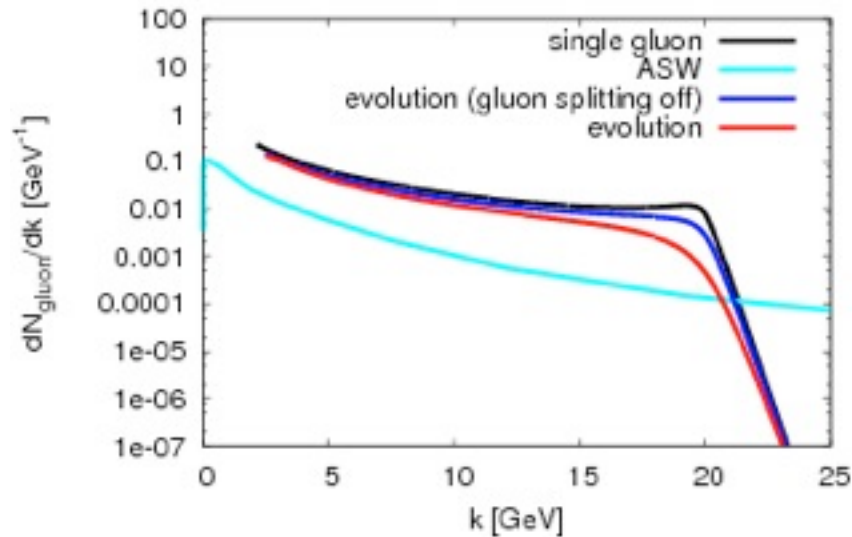


Finite-L effects:  
Vacuum-medium interference  
+ large-angle cut-off

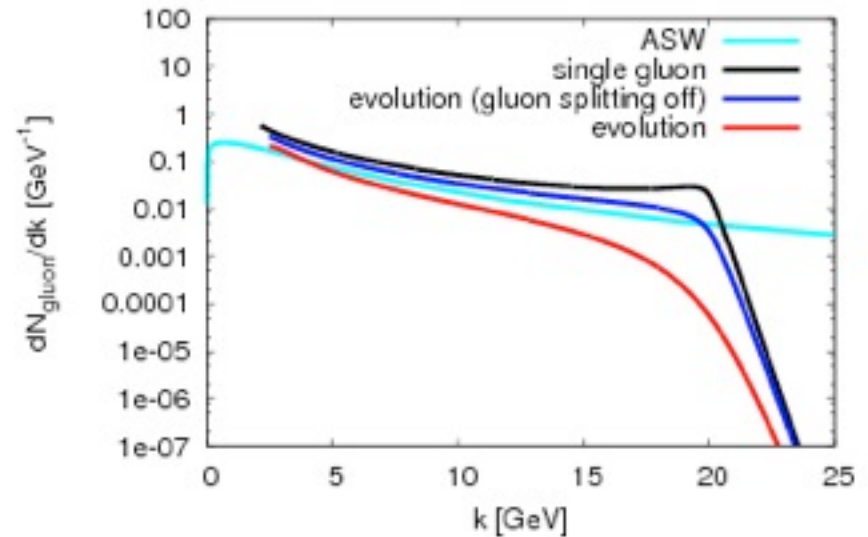
# AMY and BDMPS



$p=20$  GeV,  $T=300$  MeV,  $L=2$ fm



$p=20$  GeV,  $T=300$  MeV,  $L=5$ fm



Large difference between AMY and ASW at  $L=2$  fm?

# HT and GLV

Single-gluon kernel GLV and HT 'similar'

$$\text{HT: } \frac{dN}{dx dk_T^2} = \frac{\alpha_s C_F}{\pi} P_{qg}(x) F_{gg} \frac{1}{k_T^4} \Omega$$

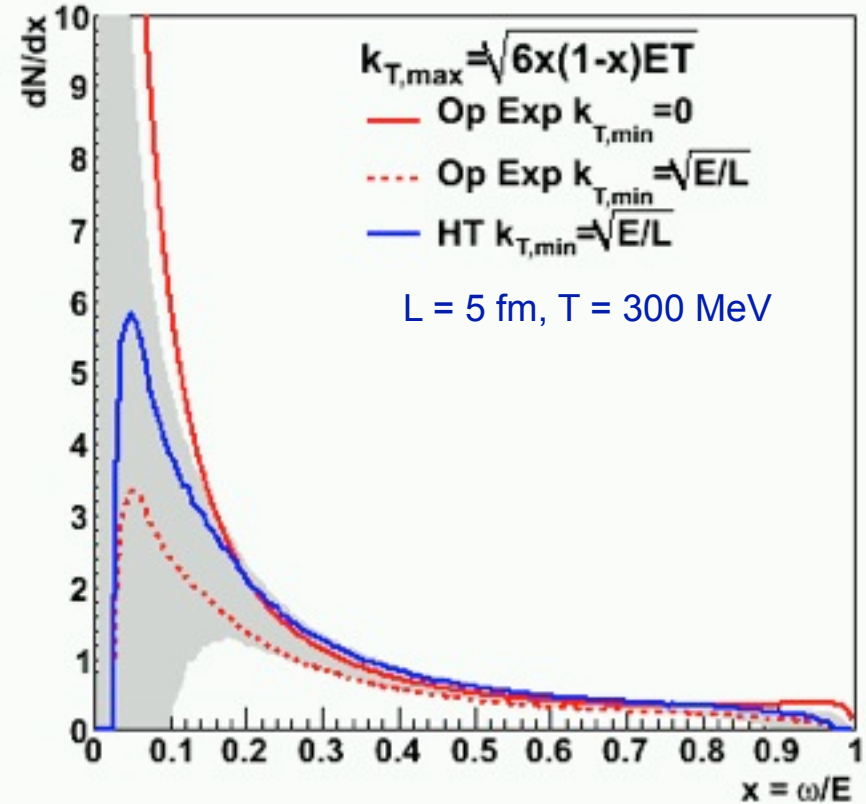
$$\text{OE: } \frac{dN}{dx dk_T^2} \approx \frac{\alpha_s C_F}{\pi} P_{qg}(x) \frac{1}{\lambda} \frac{\mu^2}{(k_T^2 + \mu^2)^2} \Omega$$

$$\Omega = \int_0^L d\xi \left[ 1 - \cos\left( \frac{k_T^2 \xi}{2 p z (1-z)} \right) \right]$$

HT:  $\tau < L \Rightarrow k_T > \sqrt{(E/L)}$   
kernel diverges for  $k_T \rightarrow 0$

$$\text{HT: } k_{T,\max} = \sqrt{2 x (1-x) E 3 T}$$

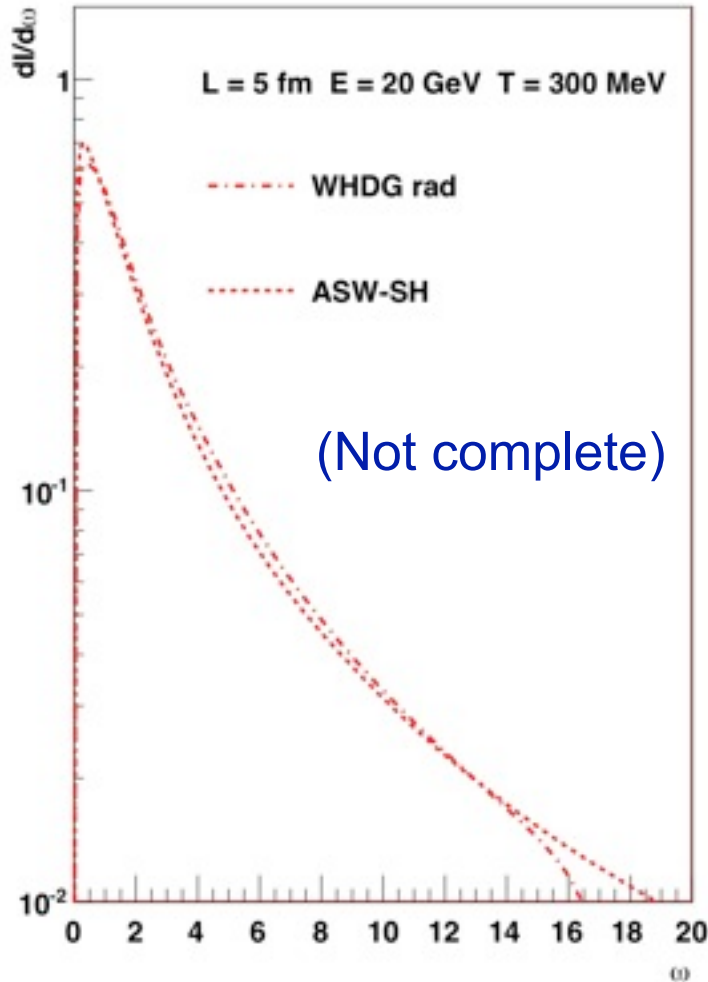
$$\text{GLV: } k_{T,\max} = \sqrt{2 x (1-x) E} \quad q_{T,\max} = \sqrt{3 E T}$$



Large uncertainty from  $k_{T,\max}$

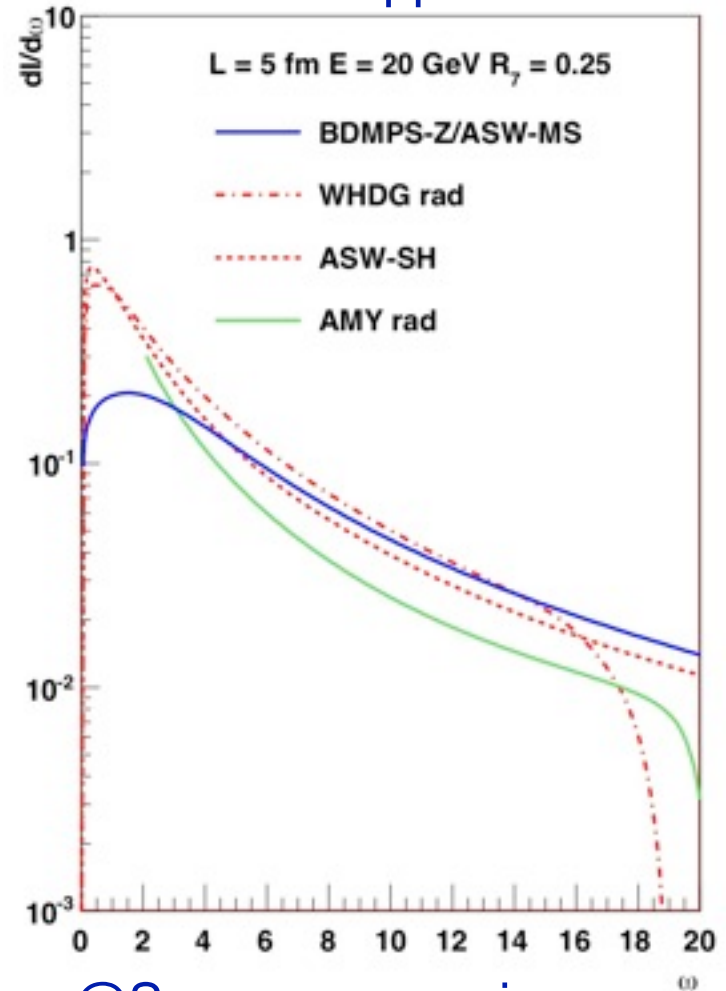
# Single gluon spectra

Same temperature



@Same temperature:  
 AMY > OE > ASW-MS

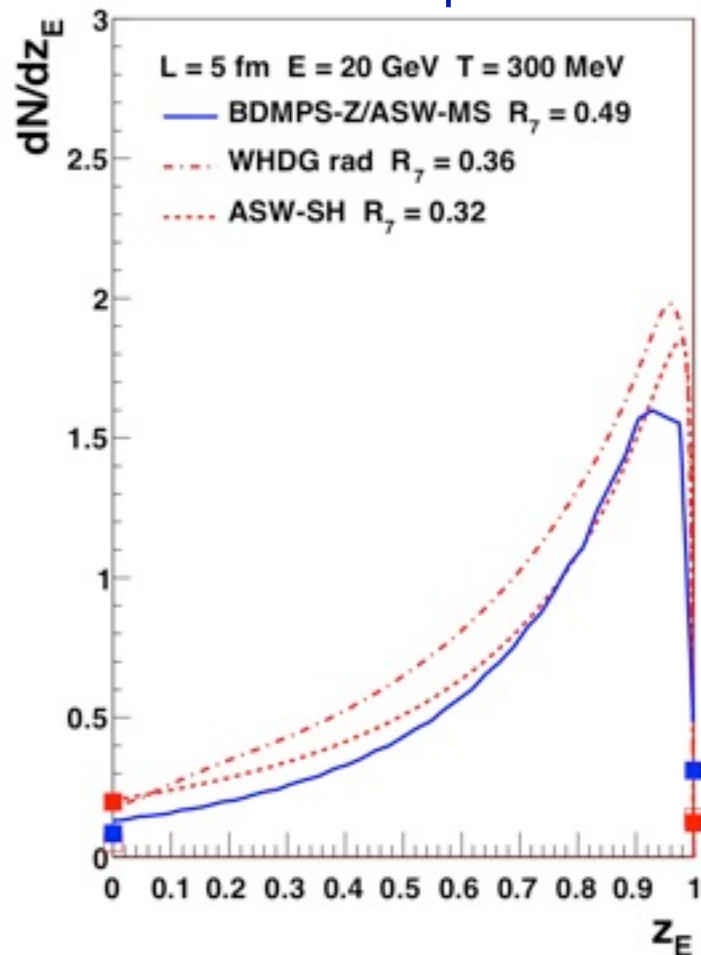
Same suppression



@Same suppression:  
 OE (AMY?) peaked at low  $\omega$   
 ASW-MS not so much

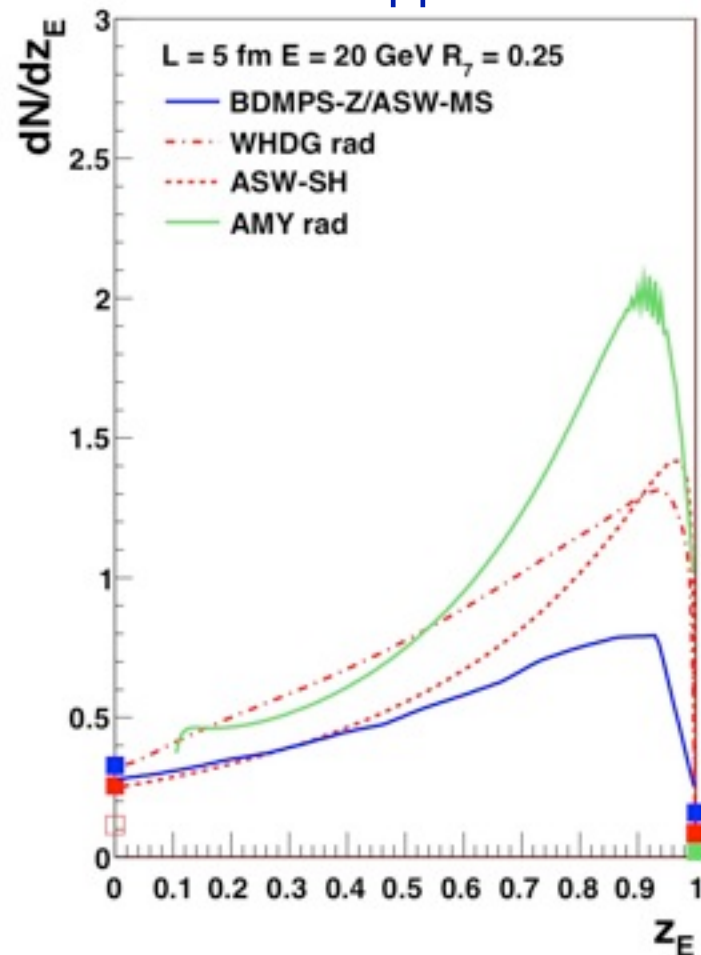
# Outgoing quark spectra

Same temperature



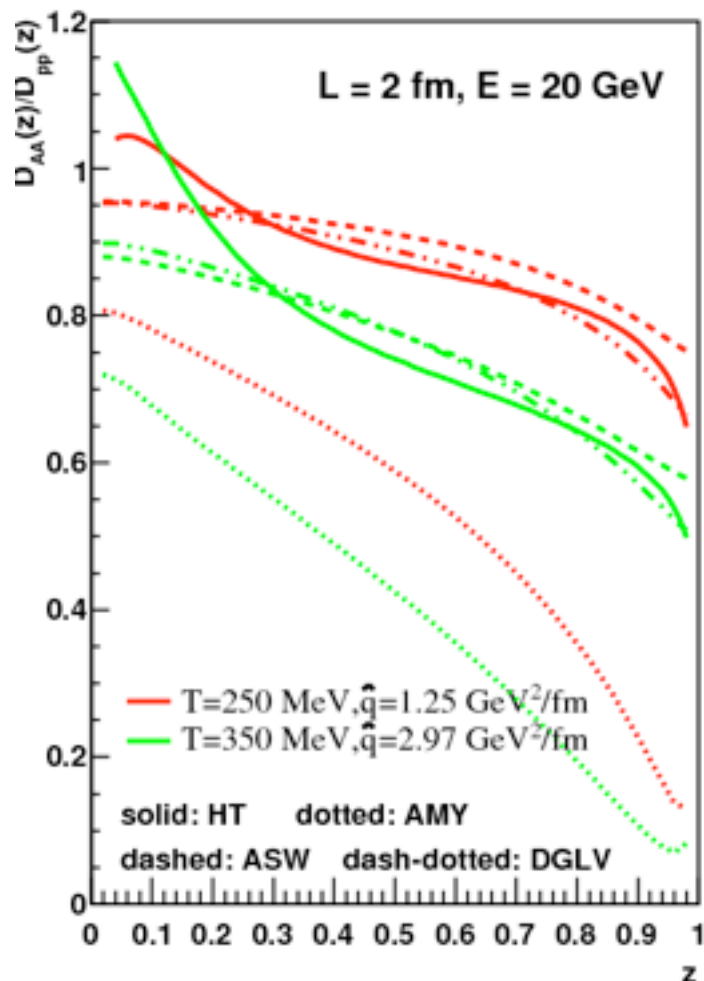
ASW-MS less suppression than OE at  $T=300 \text{ MeV}$

Same suppression

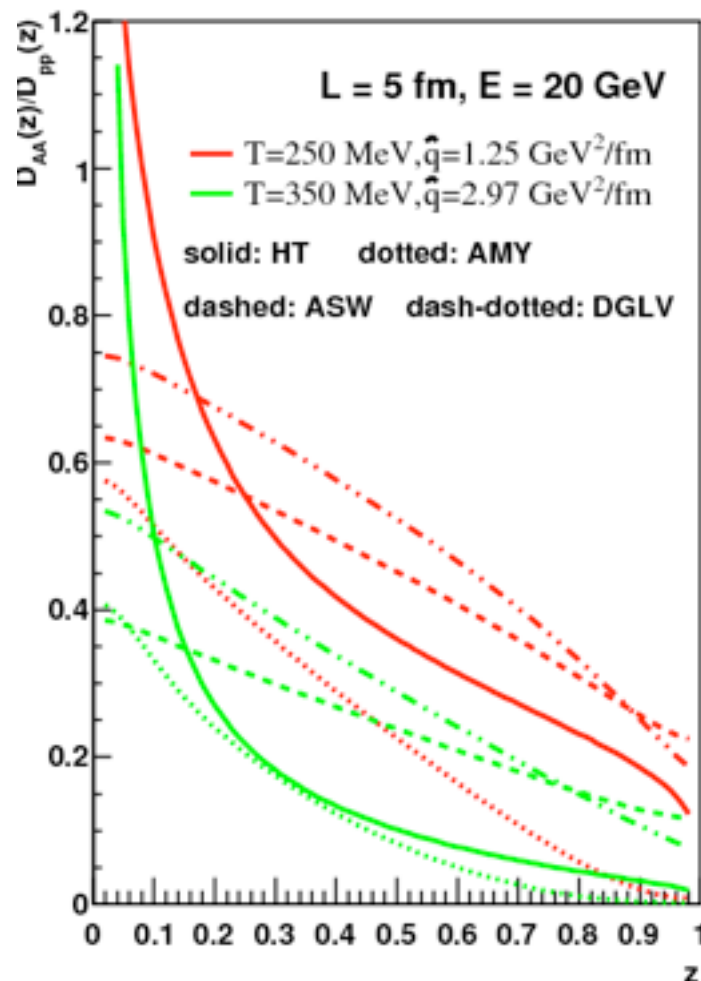


At  $R_7 = 0.25$   
 $P_0$  small for ASW-MS  
 $P_0 = 0$  for AMY by definition

# Fragmentation function



L=2 fm, T=250, 350 MeV  
 GLV, HT, ASW-MS similar  
 AMY: large suppression

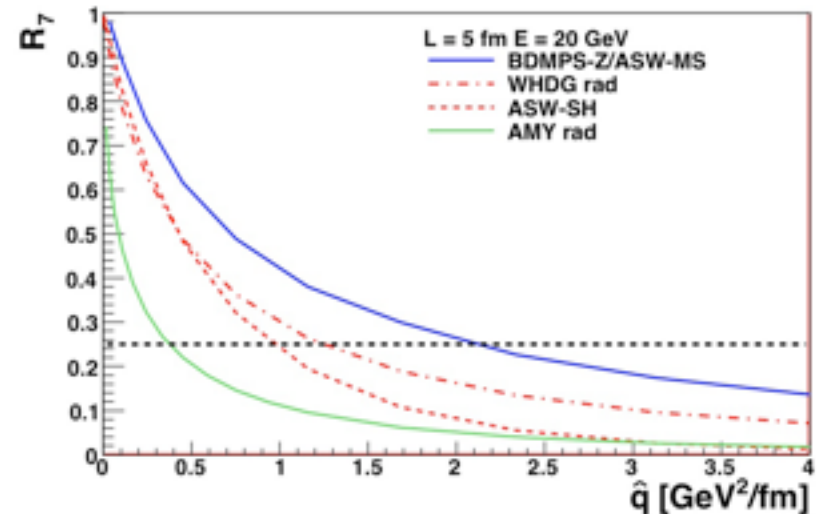


L=2 fm, T=250, 350 MeV  
 AMY, HT larger suppression  
 than OE, MS



# Conclusion

- Tentative summary:
  - AMY shows strongest suppression  
Lack of vacuum radiation?
  - ASW-MS: smallest suppression  
Soft scattering or interference or both?
  - OE, HT similar, between MS and AMY
- Large uncertainties associated with large angle radiation in all formalisms
- Differences between formalisms large at single-gluon level  
 $R_{AA}$  probably not sensitive to details of multi-gluon treatment



In preparation: TECHQM publication with more detailed report

Thanks to all in TECHQM who contributed !

**Extra slides**

# $X_+$ vs $xE$

