Tranverse momentum broadening and the jet quenching parameter, Redux

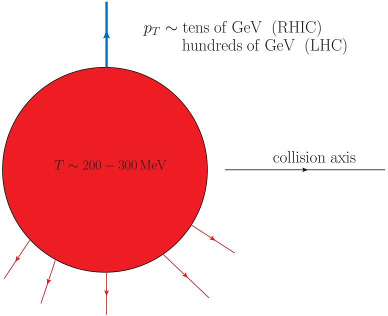
based on: FDE, H. Liu and K. Rajagopal, arXiv:1006.1367

Francesco D'Eramo

Massachusetts Institute of Technology

LBL - JET Summer School 2010

Introduction and Motivation



High p_T hadrons suppression

Medium produced in the RHIC collisions is able to quench jets. Occasionally the hard valence partons undergo hard scattering, two back-to-back hard partons with a large p_T in the final state.

High p_T partons as probes

- the parton propagates as much as $5 - 10\,\mathrm{fm}$ within the medium
- production cross-sections for hard partons well known (both by pQCD and data from proton-nucleus collisions)

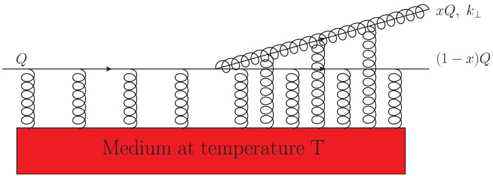
The medium has two main effects on the propagating hard parton:

- changing direction of its momentum (transverse momentum broadening)
- parton energy loss

Energy loss in the high energy limit

Radiative energy loss

In the high energy limit enegy loss dominated by the QCD analogue of bremsstrahlung.



The incident and outgoing partons and the radiated gluon are constantly kicked by the medium: they are all subjects to transverse momentum broadening.

The jet quenching parameter $\hat{q}$

The jet quenching parameter is defined as the mean transverse momentum picked up by the hard parton per unit distance travelled (or in the high energy limit unit time)

$$\hat{q} = \frac{\langle k_\perp^2 \rangle}{L}, \qquad (L = \text{medium length})$$

The jet-quenching parameter $\hat{q}$ plays a central role in the energy loss calculation, but it is defined via transverse momentum broadening only.

Toward a factorized description

Separation of scales

Energy loss and transverse momentum broadening involve widely separated scales

$$Q \gg l_\perp \gg T$$

Factorized description physics at each scale cleanly separated at lowest nontrivial order, correction to factorization systematically calculable, order by order in the small ratio between the scales.

First step

Formulation of the momentum broadening in the language of Soft Collinear Effective Theory (SCET).

Our focus

Non-radiative $k_\perp$ broadening in the $Q \to \infty$ limit:

- easiest case to handle;
- natural context in which the jet quenching parameter arises.

Our language: SCET

In the $T \ll Q$ limit:

- natural separation of scales
- natural organization of the modes into kinematic regimes

Set-up of the problem

Energy scales

Study propagation of a hard parton with initial four momentum

$q_0 \equiv (q_0^+, q_0^-, q_{0\perp}) = (0, Q, 0)$

propagating through some form of QCD matter. Consider QGP in equilibrium at temperature T (although our analysis would apply to other forms of matter).

We assume $Q \gg T$, we have a small dimensionless ratio $\lambda \equiv \frac{T}{Q} \ll 1$.

Goal

Characterize the transverse momentum broadening by computing $P(k_\perp)$, the probability distribution for the hard parton to acquire transverse momentum $k_\perp$ after traversing the medium.

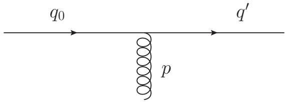
$P(k_\perp)$ depends on the medium length L.

Normalization convention $\int \frac{d^2 k_\perp}{(2\pi)^2} P(k_\perp) = 1$

SCET Lagrangian and relevance of Glauber gluons

Idilbi, Majumder, Phys.Rev.D80:054022,2009. [arXiv:0808.1087]

$q_0 = (0, Q, 0) \qquad q' = q_0 + p$



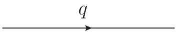
Glauber gluon: $p = (\lambda^2, \lambda^2, \lambda)Q$

Final state collinear parton: $q' = Q(\lambda^2, 1, \lambda)$
Further Glaubers keep the part off-shell by the same order, $q'^2 \sim \lambda^2 Q^2$, not induced radiation
Interaction vertex: $\alpha_s(T)$

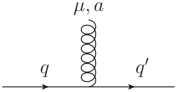
SCET Lagrangian at $\mathcal{O}(\lambda^4)$

Interaction between collinear parton and Glauber gluons.
Power counting in λ at the level of the Lagrangian.
At the leading order in λ ($\mathcal{O}(\lambda^4)$)

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp} \, \bar{\xi}_{\bar{n}, q'_\perp} \left[i\bar{n}\cdot D + \frac{q_\perp^2}{2Q} \right] \not{n} \xi_{\bar{n}, q_\perp}$$



$= i \, \not{\bar{n}} \frac{Q}{2q^+ Q - q_\perp^2 + i\epsilon}$

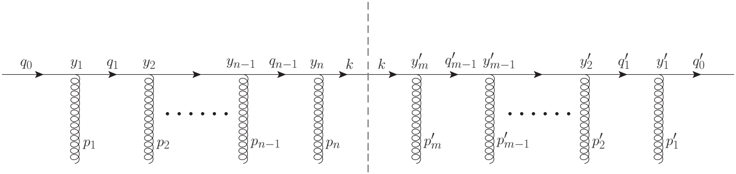


$= i g t^a n_\mu \, \not{\bar{n}}$

Relating $P(k_\perp)$ to the S-matrix

Strategy

- Probability amplitude for the process $\alpha \to \beta$: $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$;
- In our set-up β differs from α only on its value of $k_\perp$,;
- The S-matrix is unitary: $\sum_\beta \left|S_{\beta\alpha}\right|^2 = 1 \Rightarrow 2\,\mathrm{Im}\; M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$;
- Compute $2\,\mathrm{Im}\; M_{\alpha\alpha}$ by cutting diagrams, use the unitarity relation to identify $|M_{\beta\alpha}|^2$;
- Evaluate $P(k_\perp)$ for $k_\perp \neq 0$;
- The normalization condition $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$ fixes $P(0)$;
- Leading order in λ contribution: $2\,\mathrm{Im}\,M_{\alpha\alpha} = \sum_{m=1,n=1}^{\infty} \mathcal{A}_{mn} = \sum_{m=1,n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$



A few comments on $\frac{d^2 \mathcal{A}_{mn}}{d^2 k_\perp}$

The cut momentum $k_\perp$

The cut momentum k is the four-momentum of the hard parton in the final state.

For forward scattering amplitude: $q_0 = q'_0 \;\Rightarrow\; k_\perp = \sum_{i=1}^{n} p_{i\perp} = \sum_{i=1}^{m} p'_{i\perp}$

$p_{i\perp} \sim p'_{i\perp} \sim T$, $k_\perp$ may be larger. Typical value $k_\perp^2 \sim \hat{q}L$, in particular $k_\perp^2$ grows with L.

A_μ as a background field

Hard parton propagation in a given field configuration $A_\mu(p)$. Nonperturbative physics of the medium does not enter this calculation. Average over configurations at the end.

Amplitude in the $Q \to \infty$ limit: $Q \gg k_\perp^2 L \sim \hat{q}L^2$

If this condition is satisfied the propagators of the internal quarks are $\propto \delta^2(z_\perp)$.
It requires L is short enough that the hard parton trajectory in position space remains well-approximated as a straight line, even though it picks up transverse momentum.

Final result for $P(k_\perp)$

Expression for $P(k_\perp)$

$$P(k_\perp) = \int d^2x_\perp \, e^{ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp), \qquad \mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr} \left[W^\dagger_\mathcal{R}[0, x_\perp] \, W_\mathcal{R}[0, 0] \right] \right\rangle$$

where

$$W_\mathcal{R}[0, x_\perp] \equiv P \left\{ \exp \left[ig \int_0^{L^-} dx^- \, A^+_\mathcal{R}(0, x^-, x_\perp) \right] \right\}$$

for a collinear particle in the $SU(N)$ representation $\mathcal{R}$, with dimension $d(\mathcal{R})$.

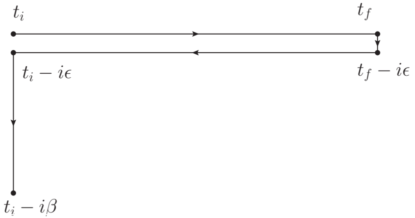
Properties of $P(k_\perp)$

- $P(k_\perp)$ depends only on the medium property (thus also $\hat{q}$ does).
- Transverse momentum broadening without radiation: field theoretically well-defined property of the medium.
- This is the kind of factorization we hope to find once radiation is included.

$\hat{q}$ from light-like Wilson lines

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{\sqrt{2}}{L^-} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \int d^2 x_\perp \, e^{i k_\perp \cdot x_\perp} \mathcal{W}_\mathcal{R}(x_\perp)$$

Recall $A^+ = (A^+)^a t^a$. In $\mathcal{W}_\mathcal{R}(x_\perp)$ both $(A^+)^a$ and t^a are path ordered.



$\mathcal{W}_\mathcal{R}(x_\perp)$ should be described using the Schwinger-Keldysh contour

- one of the light-like Wilson lines on the $\mathrm{Im}\, t = 0$ segment
- the other light-like Wilson line on the $\mathrm{Im}\, t = -i\epsilon$ segment

$\hat{q}$ evaluation in $\mathcal{N} = 4$ SYM revisited

Distinction in ordering not made in the AdS/CFT evaluation (Liu, Rajagopal, Wiedemann, PR97,2006 [hep-ph/0605178]).
Ordering by Lorentzian AdS/CFT (Skenderis, van Rees, JHEP 0905:085,2009. [arXiv:0812.2909]).

Ordering subtlety do not change the result: $\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} \sqrt{g_{YM}^2 N_c}\, T^3$

Summary and future directions

Summary

- Probability distribution $P(k_\perp)$ evaluation within an EFT formalism;
- Glaubers responsible for $k_\perp$ broadening in the absence of radiation;
- $P(k_\perp)$ and $\hat{q}$ depend on the medium property only (factorization);
- Subtleties about the operators ordering: strong coupling $\hat{q}$ evaluation more straightforward, previous result unchanged.

Future directions

- include radiation, see how $\hat{q}$ enters in the spectrum of the radiated gluons;
- include higher order corrections in λ;
- weak-coupling $\hat{q}$ evaluation for QCD plasma at high enough T;
- compare our $P(k_\perp)$ with the correspondent quantity in $N = 4$ SYM.