Estimating Uncertainties on Quarkonium Production in the Color Evaporation Model

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Color Evaporation Model

All quarkonium states are treated like $Q\overline{Q}$ ($Q = c, b$) below $H\overline{H}$ ($H = D, B$) threshold

Distributions for all quarkonium family members similar, modulo decay feed down, production ratios should be independent of $\sqrt{s}$

At LO, $gg \to Q\overline{Q}$ and $q\overline{q} \to Q\overline{Q}$; NLO add $gq \to Q\overline{Q}q$

$$\sigma_Q^{\rm CEM} = F_Q \sum_{i,j} \int_{4m_Q^2}^{4m_H^2} d\hat{s} \int dx_1 dx_2 \; f_{i/p}(x_1, \mu^2) \; f_{j/p}(x_2, \mu^2) \; \hat{\sigma}_{ij}(\hat{s}) \; \delta(\hat{s} - x_1 x_2 s)$$

Values of m_Q and Q^2 fixed from NLO calculation of $Q\overline{Q}$ production

Main uncertainties arise from choice of PDFs, heavy quark mass, renormalization (α_s) and factorization (evolution of PDFs) scales

Inclusive F_Q fixed by comparison of NLO calculation of $\sigma_Q^{\rm CEM}$ to $\sqrt{s}$ dependence of J/ψ and Υ cross sections, $\sigma(x_F > 0)$ and $B d\sigma/dy|_{y=0}$ for J/ψ, $B d\sigma/dy|_{y=0}$ for Υ

Data and branching ratios used to separate the F_Q's for each quarkonium state

Resonance	J/ψ	ψ'	χ_{c1}	χ_{c2}	Υ	Υ'	Υ''	$\chi_b(1P)$	$\chi_b(2P)$
$\sigma_i^{\rm dir}/\sigma_H$	**0.62**	**0.14**	**0.6**	**0.99**	**0.52**	**0.33**	**0.20**	**1.08**	**0.84**
f_i	**0.62**	**0.08**	**0.16**	**0.14**	**0.52**	**0.10**	**0.02**	**0.26**	**0.10**

Table 1: **The ratios of the direct quarkonium production cross sections, $\sigma_i^{\rm dir}$, to the inclusive J/ψ and Υ cross sections, denoted σ_H, and the feed down contributions of all states to the J/ψ and Υ cross sections, f_i, Digal *et al.*.**

Why Still CEM?

Open and hidden charm photo- and hadroproduction show similar energy dependence

High p_T Tevatron Run I data show that, within uncertainties of the data, the prompt J/ψ, the ψ' and χ_c p_T dependencies are the same

Amundsen *et al.* calculated p_T distribution (only partial real part) harder than data at high p_T, undershoots at low p_T – likely because they do not include any k_T smearing

Gavai *et al.* calculated complete J/ψ p_T distribution starting from exclusive NLO $Q\overline{Q}$ production code by Mangano *et al.*

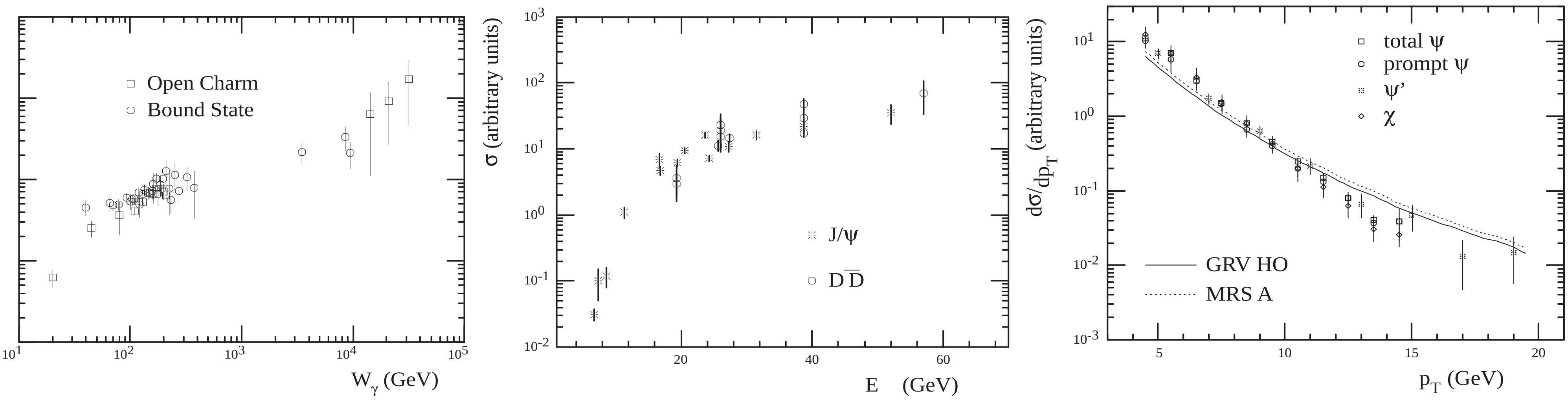


Figure 1: **(Left) Photoproduction data as a function of the photon energy in the hadron rest frame, W_γ. (Center) Hadroproduction data as a function of the center-of-mass energy, E_{cm}. In both cases, the normalization has been adjusted to show the similar shapes of the data. (Right) Run I data from the CDF Collaboration, shown with arbitrary normalization. The curves are the predictions of the color evaporation model at tree level, also shown with arbitrary normalization. [Amundson *et al.*]**

How to Fix the Uncertainty on the CEM Result?

Previously took 'by eye' fit to $Q\overline{Q}$ total cross section

Dates back to original Hard Probes Collaboration report in 1995 – only PDF changed over time

Since I've been asked what the uncertainty on the cross section is, I have to try to invent some, this is that attempt

One final remark: there is no calculation of the polarization, would need to start from NLO polarized $Q\overline{Q}$ production calculation

BTW, no prediction does not necessarily mean a flat distribution, it means there is no calculation

Choosing J/ψ Parameters I: FONLL-based

Main sources of uncertainty:

Mass: $1.3 < m < 1.7$ **GeV for charm (central value, 1.5 GeV)**

Scale: renormalization, μ_R, and factorization, μ_F, scales governing α_s and PDF behavior respectively

Parton Density: evolution of gluon density

With a given PDF set define a fiducial region of mass and scale that should encompass the true value:

- **For $\mu_F = \mu_R = m$, vary mass between upper and lower end of range;**
- **For central mass value, vary scales independently within a factor of two: $(\mu_F/m, \mu_R/m) = (1,1)$, (2,2), (0.5,0.5), (0.5,1), (1,0.5), (1,2), (2,1).**

Define upper and lower bounds of theoretical values; the maximum and minimum may not come from the same set of parameters at a given energy or p_T

The uncertainty band comes from the upper and lower limits of mass and scale uncertainties added in quadrature:

$$\sigma_{\rm max} = \sigma_{\rm cent} + \sqrt{(\sigma_{\mu,{\rm max}} - \sigma_{\rm cent})^2 + (\sigma_{m,{\rm max}} - \sigma_{\rm cent})^2}$$

$$\sigma_{\rm min} = \sigma_{\rm cent} - \sqrt{(\sigma_{\mu,{\rm min}} - \sigma_{\rm cent})^2 + (\sigma_{m,{\rm min}} - \sigma_{\rm cent})^2}$$

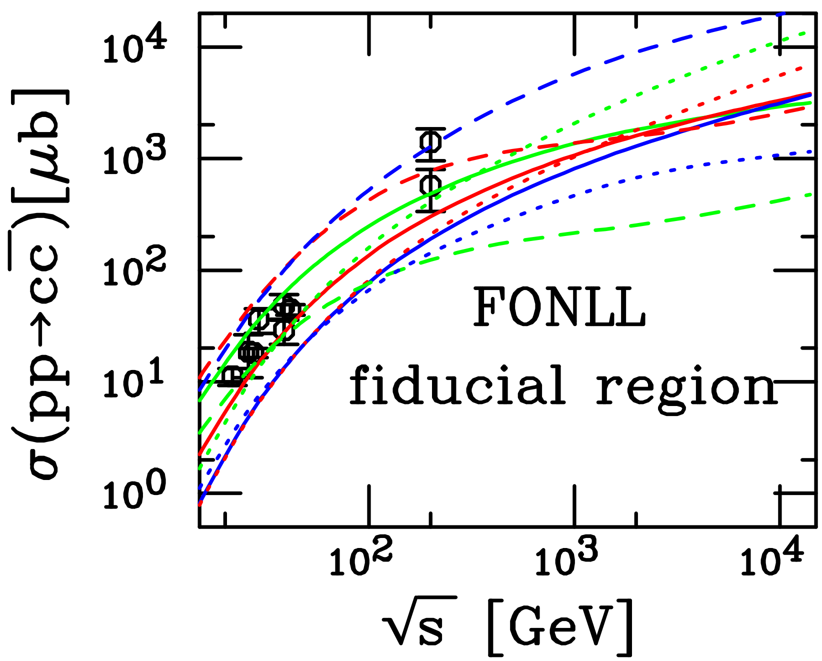
FONLL Calculation of $c\overline{c}$ Uncertainty

$c\overline{c}$ cross section dependence on $\sqrt{s}$ with FONLL parameter sets (left), uncertainty band on $c\overline{c}$ cross section (right)

None of the FONLL sets fit the data, large χ^2/dof

No convergence for $\mu_R/m < 1$ (large α_s)

Problems with backward evolution of PDFs for $\mu_F/m \leq 1$ (near or below minimum scale of PDFs)



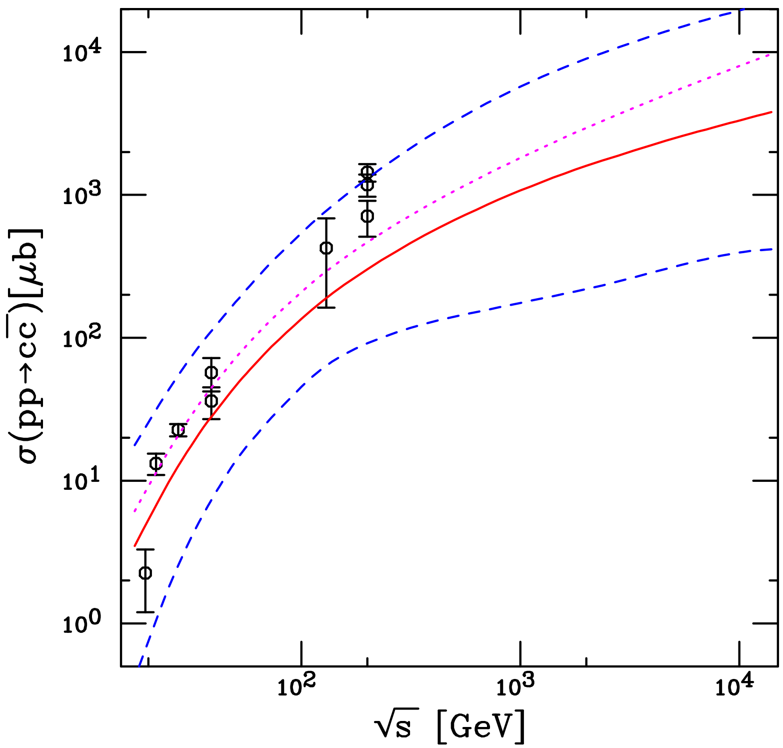


Figure 2: (Left) Total $c\overline{c}$ cross sections calculated using CTEQ6M. The solid red curve is the central value $(m, \mu_F/m, \mu_R/m) = (1.5\,\text{GeV}, 1, 1)$. The green and blue solid curves are $(1.3\,\text{GeV}, 1, 1)$ and $(1.7\,\text{GeV}, 1, 1)$ respectively. The red, blue and green dashed curves correspond to $(1.5\,\text{GeV}, 0.5, 0.5)$, $(1.5\,\text{GeV}, 1, 0.5)$ and $(1.5\,\text{GeV}, 0.5, 1)$ while the red, blue and green dotted curves are for $(1.5\,\text{GeV}, 2, 2)$, $(1.5\,\text{GeV}, 1, 2)$ and $(1.5\,\text{GeV}, 2, 1)$. (Right) Uncertainty band formed from adding mass and scale uncertainties in quadrature.

J/ψ Uncertainty Large, Can Only Define Upper Limit

Fit F_C CEM parameter for central mass and scale value, use same value for other calculations of fiducial range

At large $\sqrt{s}$ $(\mu_F/m, \mu_R/m) = (0.5, 0.5)$, (0.5,1) flattens because $\mu_F < \mu_0$ of PDF

$m_c = 1.7$ GeV governs uncertainty at low $\sqrt{s}$ since $m_D/m_c \sim 1.1$, small phase space for J/ψ production in CEM – doesn't make much sense

Large combination of mass and scale uncertainty makes lower limit ill defined

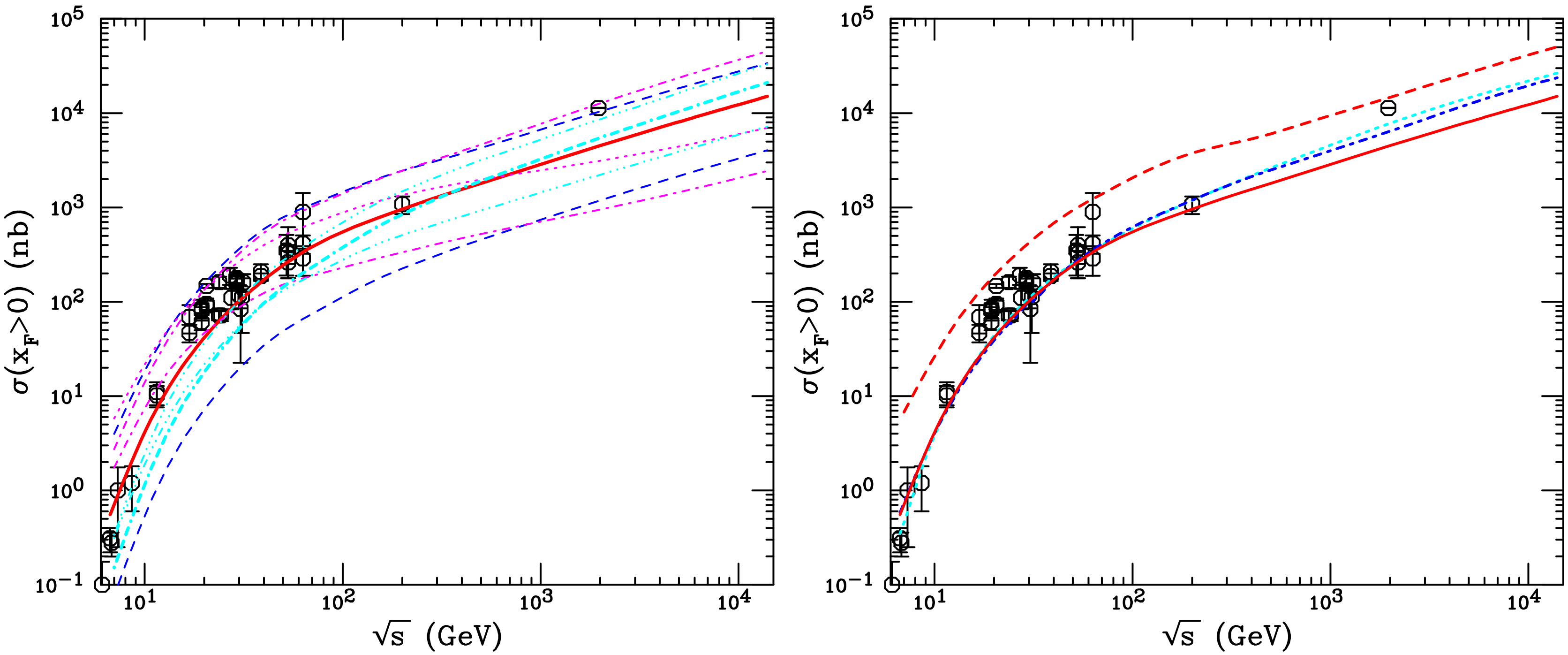


Figure 3: **(Left) Total J/ψ cross sections calculated using CTEQ6M. The solid red curve is the central value $(\mu_F/m, \mu_R/m) = (1, 1)$ with $m = 1.5$ GeV. The upper and lower dashed blue curves are $m = 1.3$ and 1.7 GeV with (1,1) respectively. The dotted magenta curve corresponds to (0.5,0.5) while the upper and lower magenta dot-dashed curves (above $\sqrt{s} = 50$ GeV) correspond to (1,0.5) and (0.5,1). The dash-dash-dotted cyan curve curresponds to (2,2) while the upper and lower cyan dot-dot-dot-dashed curves (above $\sqrt{s} = 50$ GeV) are (2,1) and (1,2). The last 6 curves are all calculated for $m_c = 1.5$ GeV. (Right) The solid and dashed red curves are the central value and upper limit for the J/ψ cross section. The solid cyan curve employs the MRST HO distributions while the dot-dashed blue curve is a result with CTEQ6M, both employing $m_c = 1.2$ GeV, $(\mu_F/m_T, \mu_R/m_T) = (2, 2)$.**

Choosing J/ψ Parameters II: Fitting $\sigma_{c\overline{c}}$

J/ψ parameters based on fits to NLO total $c\overline{c}$ cross section – caveat: full NNLO cross section unknown, could still be large correction

Fix $\mu_F/m = 1, 2$ and let μ_R/m float for range of charm quark masses, $1.1 < m < 1.5$ GeV, used too small quark masses to try to see if a minimum χ^2 has been found – do not go to higher values of m to avoid $m > m_{J/\psi}/2$

$m = 1.27$ GeV is value of charm quark mass from lattice calculations at $m(3\,\mathrm{GeV})$

Calculate χ^2/dof for fixed-target data alone as well as with RHIC, check behavior at higher energies, up to LHC

Take best fit values and use these to obtain $c\overline{c}$ cross section below $D\overline{D}$ threshold, find F_c for each mass, scale combination from fit to J/ψ data at $x_F > 0$, extrapolate to LHC energies

Also playing same game with $\mu_F = \mu_R$, obtain similar values of μ_R as before but have not yet computed J/ψ result

Fitting $\sigma_{c\overline{c}}$: Fixed-Target Only

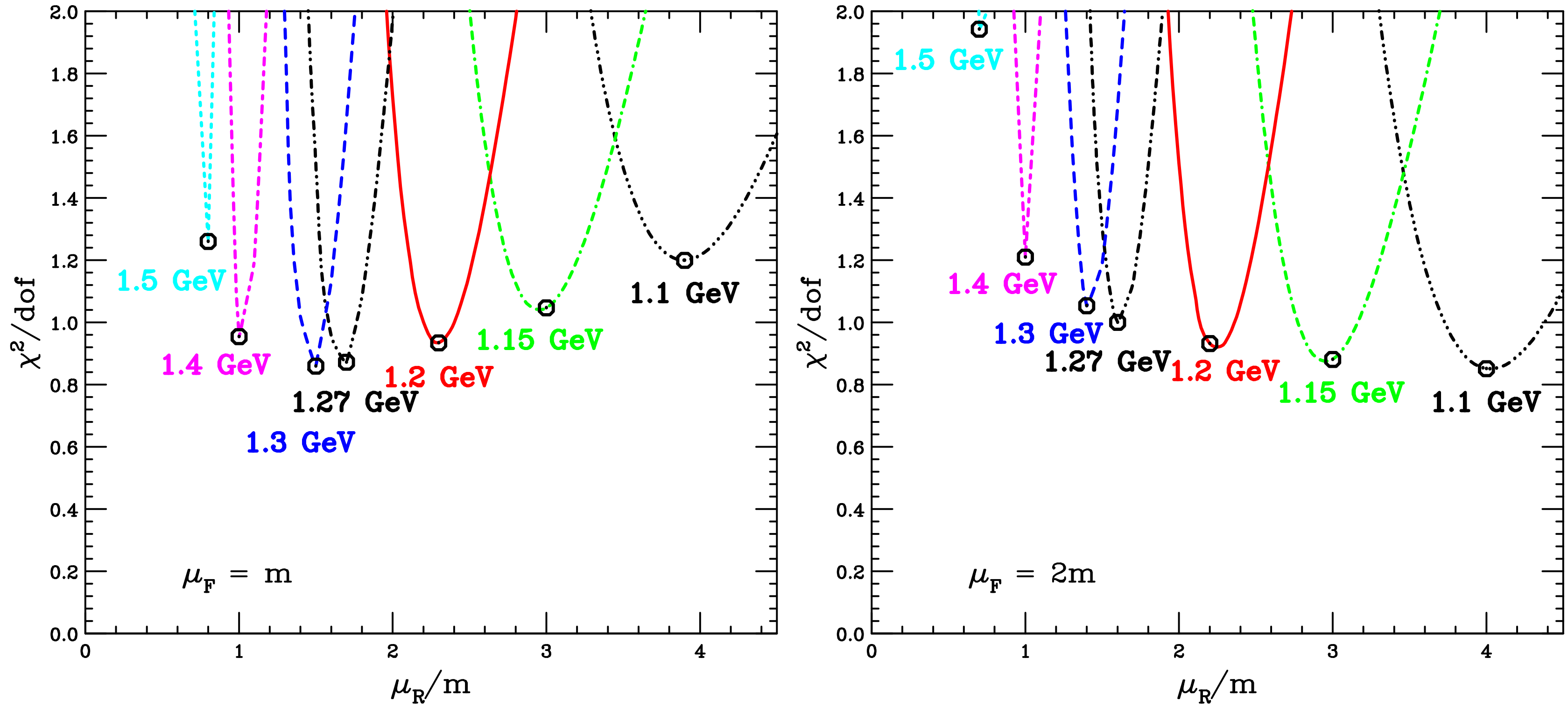


Figure 4: The calculated χ^2/dof for $\mu_F/m = 1$ (left) and 2 (right) at fixed-target energies (excluding RHIC). The circles at the minimum of the curves on the left-hand side correspond to (1.1 GeV, 1, 3.9) [dot-dot-dot-dashed black], (1.15 GeV, 1, 3) [dot-dash-dash-dashed green], (1.2 GeV, 2, 2.3) [solid red], (1.27 GeV, 1, 1.7) [dot-dot-dash-dashed black], (1.3 GeV, 1, 1.5) [dashed blue], (1.4 GeV, 1, 1) [dot-dashed magenta], and (1.5GeV, 1, 0.8) [dotted cyan] while those on the right-hand side are with (1.1 GeV, 2, 4) [dot-dot-dot-dashed black], (1.15 GeV, 2, 3) [dot-dash-dash-dashed green], (1.2 GeV, 2, 2.2) [solid red], (1.27 GeV, 2, 1.6) [dot-dot-dash-dashed black], (1.3 GeV, 2, 1.4) [dashed blue], (1.4 GeV, 2, 1) [dot-dashed magenta], and (1.5GeV, 2, 0.7) [dotted cyan]. The calculations are done with the CT10 PDFs.

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Fitting $\sigma_{c\overline{c}}$: Including RHIC

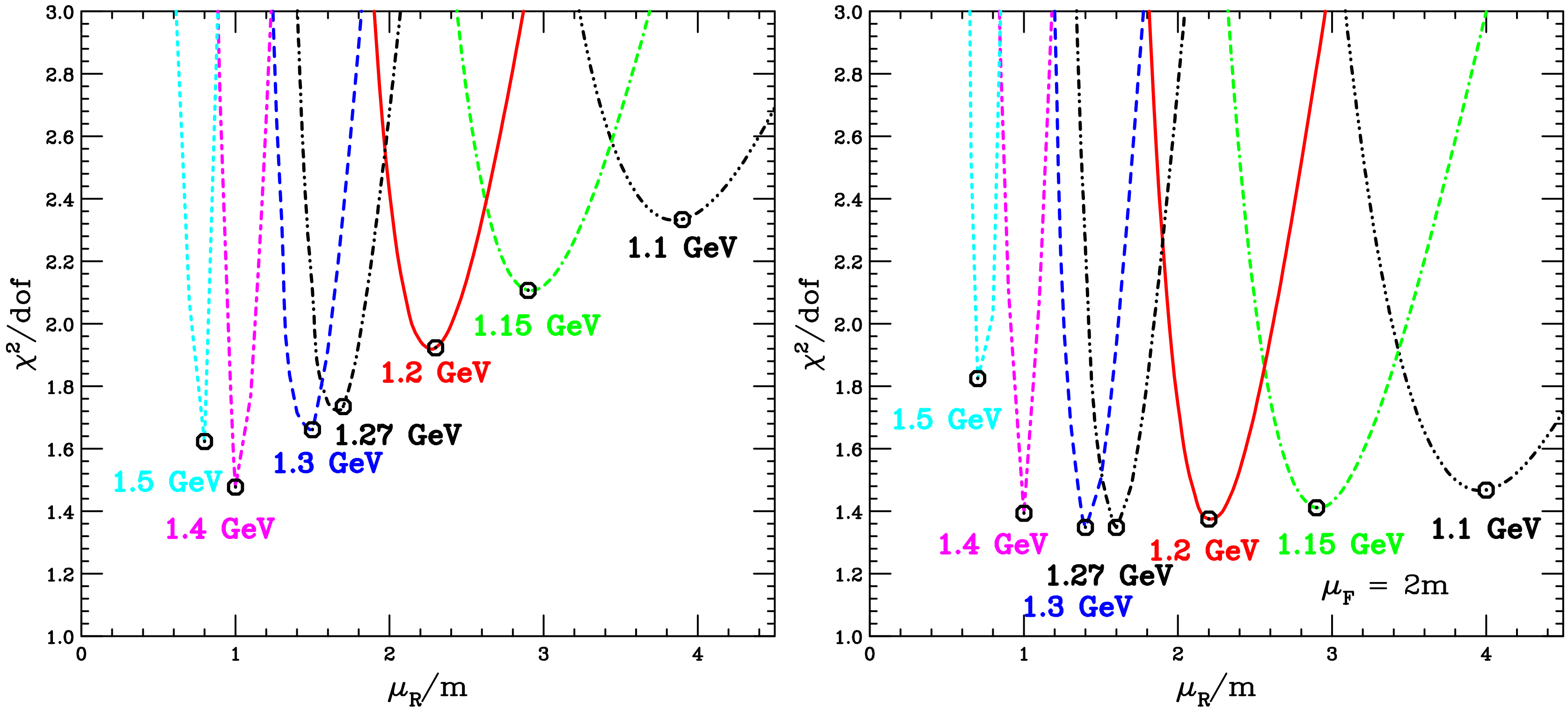

Figure 5: The calculated χ^2/dof for $\mu_F/m = 1$ (left) and 2 (right) including RHIC energies. The circles at the minimum of the curves on the left-hand side correspond to (1.1 GeV, 1, 3.9) [dot-dot-dot-dashed black], (1.15 GeV, 1, 3) [dot-dash-dash-dashed green], (1.2 GeV, 2, 2.3) [solid red], (1.27 GeV, 1, 1.7) [dot-dot-dash-dashed black], (1.3 GeV, 1, 1.5) [dashed blue], (1.4 GeV, 1, 1) [dot-dashed magenta], and (1.5GeV, 1, 0.8) [dotted cyan] while those on the right-hand side are with (1.1 GeV, 2, 4) [dot-dot-dot-dashed black], (1.15 GeV, 2, 3) [dot-dash-dash-dashed green], (1.2 GeV, 2, 2.2) [solid red], (1.27 GeV, 2, 1.6) [dot-dot-dash-dashed black], (1.3 GeV, 2, 1.4) [dashed blue], (1.4 GeV, 2, 1) [dot-dashed magenta], and (1.5GeV, 2, 0.7) [dotted cyan]. The locations of the minimum χ^2/dof do not change. The calculations are done with the CT10 PDFs.

Energy Dependence of Best $c\overline{c}$ Fits

Good agreement with fixed-target data does not guarantee good behavior at collider energies

$\mu_F/m = 2$ (right-hand side) gives more realistic $\sqrt{s}$ dependence than $\mu_F/m = 1$ (left-hand side), strongest $\sqrt{s}$ dependence with lowest μ_R/m (0.8) - largest α_s

Low masses flatten cross section for $\sqrt{s} \geq 40$ GeV due to proximity of mass to minimum scale of PDF, especially for $\mu_F = m$

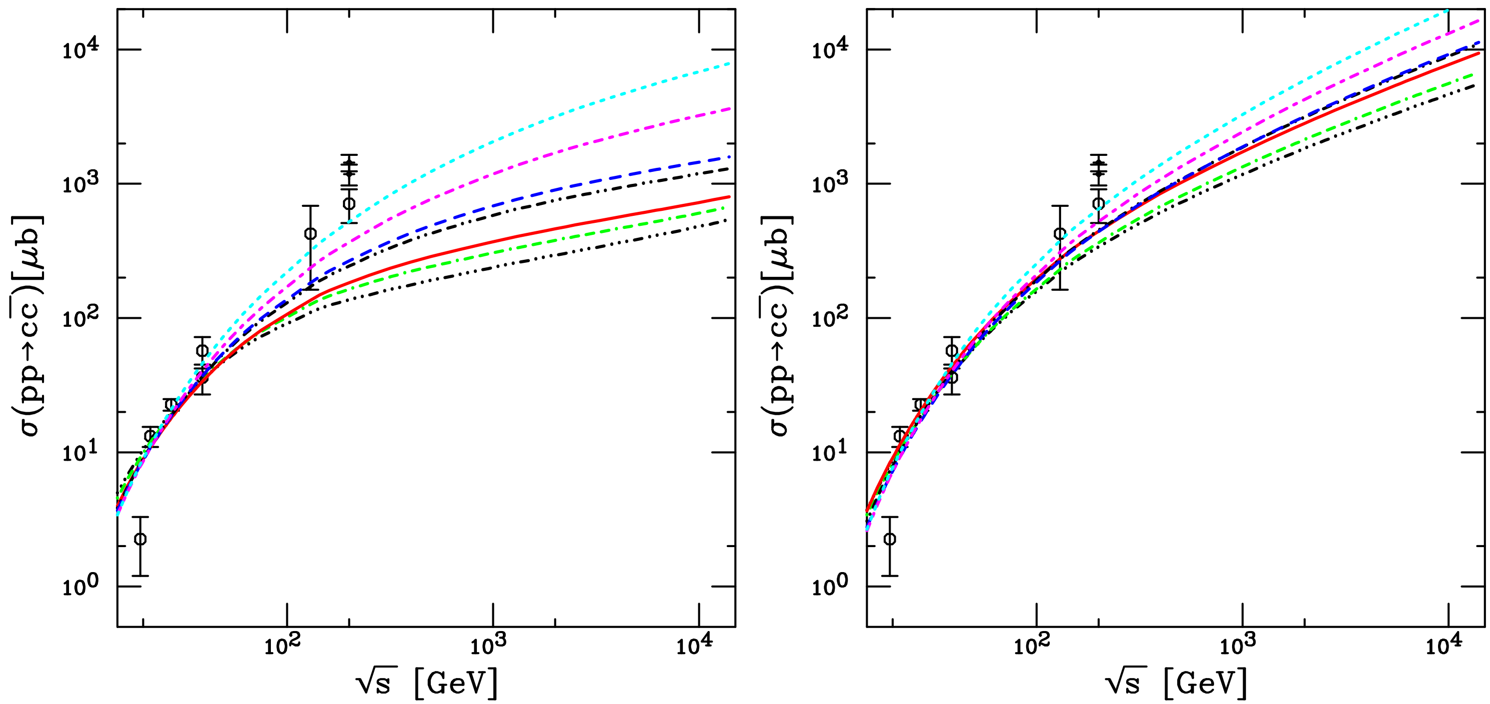

Figure 6: The calculated total $c\overline{c}$ cross sections for $\mu_F/m = 2$ (left) and 1 (right). The circles at the minimum of the curves on the left-hand side correspond to (1.1 GeV, 1, 3.9) [dot-dot-dot-dashed black], (1.15 GeV, 1, 3) [dot-dash-dash-dashed green], (1.2 GeV, 2, 2.3) [solid red], (1.27 GeV, 1, 1.7) [dot-dot-dash-dashed black], (1.3 GeV, 1, 1.5) [dashed blue], (1.4 GeV, 1, 1) [dot-dashed magenta], and (1.5GeV, 1, 0.8) [dotted cyan] while those on the right-hand side are with (1.1 GeV, 2, 4) [dot-dot-dot-dashed black], (1.15 GeV, 2, 3) [dot-dash-dash-dashed green], (1.2 GeV, 2, 2.2) [solid red], (1.27 GeV, 2, 1.6) [dot-dot-dash-dashed black], (1.3 GeV, 2, 1.4) [dashed blue], (1.4 GeV, 2, 1) [dot-dashed magenta], and (1.5GeV, 2, 0.7) [dotted cyan].

J/ψ Cross Sections from $c\overline{c}$ Fits

Take results of $c\overline{c}$ fits, calculate NLO J/ψ cross section in CEM, fit scale factor F_C

Energy dependence almost identical for $\mu_F = 2m_T$, $\sqrt{s}$ dependence generally better

CTEQ6M and CT10 have nearly same value of F_C so previous results compatible with previous results

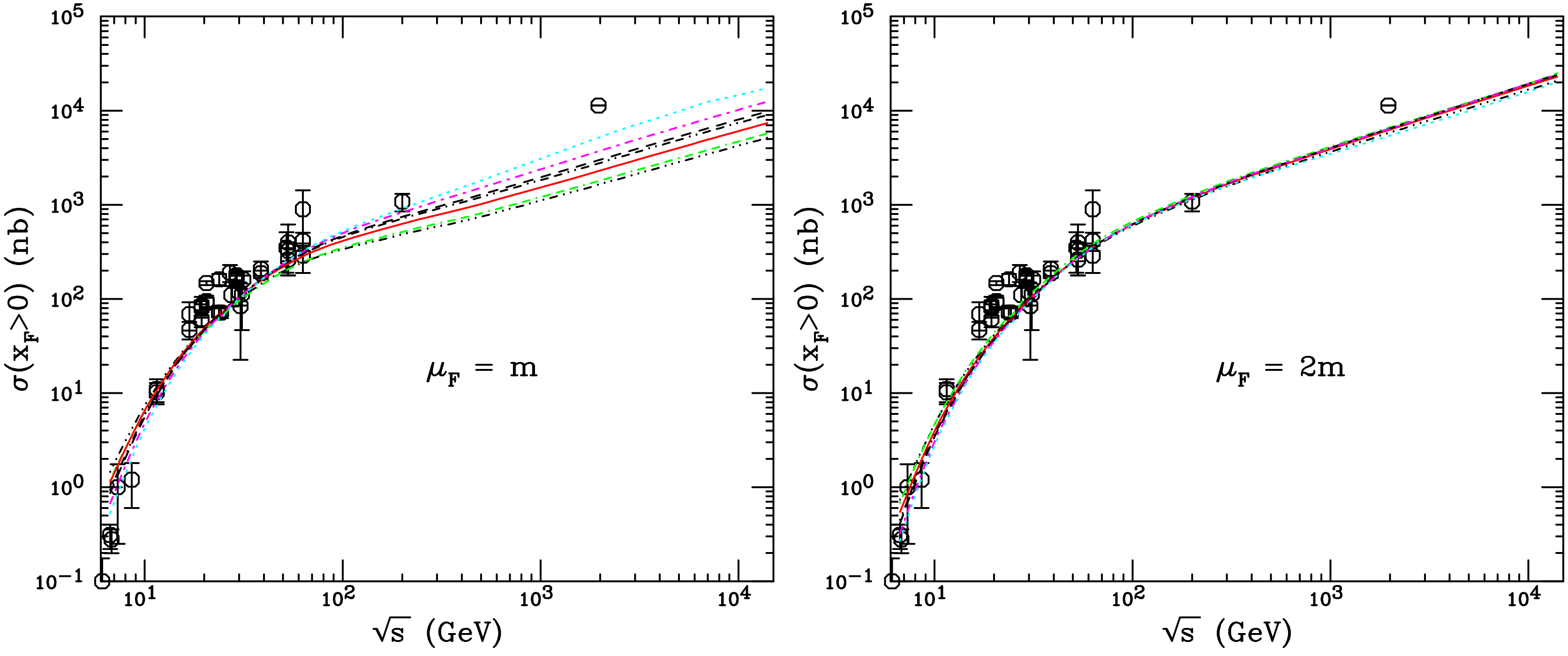

Figure 7: The calculated forward J/ψ cross sections. The curves are calculated with $(1.1\,\mathrm{GeV}, 1, 3.9)$ [dot-dot-dot-dashed black], $(1.15\,\mathrm{GeV}, 1, 3)$ [dot-dash-dash-dashed green], $(1.2\,\mathrm{GeV}, 2, 2.3)$ [solid red], $(1.27\,\mathrm{GeV}, 1, 1.7)$ [dot-dot-dash-dashed black], $(1.3\,\mathrm{GeV}, 1, 1.5)$ [dashed blue], $(1.4\,\mathrm{GeV}, 1, 1)$ [dot-dashed magenta], and $(1.5\mathrm{GeV}, 1, 0.8)$ [dotted cyan] while those on the right-hand side are with $(1.1\,\mathrm{GeV}, 2, 4)$ [dot-dot-dot-dashed black], $(1.15\,\mathrm{GeV}, 2, 3)$ [dot-dash-dash-dashed green], $(1.2\,\mathrm{GeV}, 2, 2.2)$ [solid red], $(1.27\,\mathrm{GeV}, 2, 1.6)$ [dot-dot-dash-dashed black], $(1.3\,\mathrm{GeV}, 2, 1.4)$ [dashed blue], $(1.4\,\mathrm{GeV}, 2, 1)$ [dot-dashed magenta], and $(1.5\mathrm{GeV}, 2, 0.7)$ [dotted cyan]. $(1.2\ \mathrm{GeV}, 2, 2)$ [solid red], $(1.4\ \mathrm{GeV}, 2, 1)$ [dot-dashed magenta], and $(1.5\ \mathrm{GeV}, 2, 0.7)$ [dotted cyan] using the CT10 PDFs.

Results with $\mu_F = \mu_R$ Fixed by $c\overline{c}$ Data

The χ^2 for this method is somewhat lower with RHIC included, μ is very similar to μ_R from previous fits with fixed μ_F

With fixed-target data alone, lattice charm mass gives lowest χ^2/dof

No J/ψ results yet from these fits

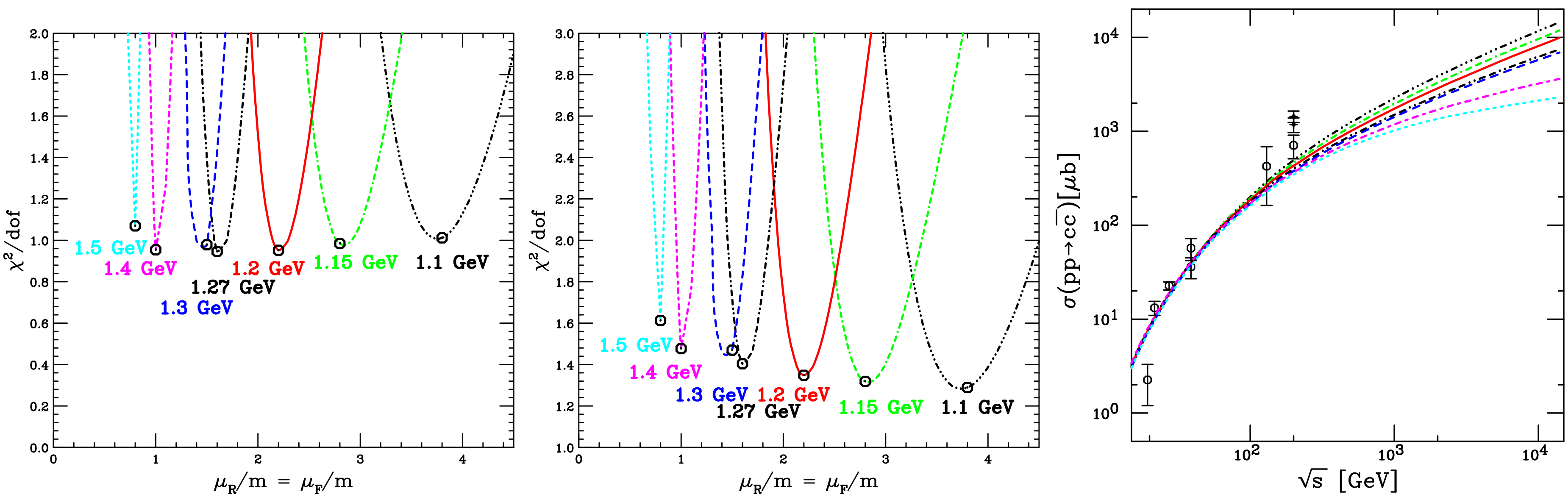

Figure 8: The calculated χ^2/dof for $\mu_F = \mu_R$ at fixed-target energies only (left) and including RHIC (middle). The circles at the minimum of the curves on the left-hand side correspond to (1.1 GeV, 3.8, 3.8) [dot-dot-dot-dashed black], (1.15 GeV, 2.8, 2.8) [dot-dash-dash-dashed green], (1.2 GeV, 2.2, 2.2) [solid red], (1.27 GeV, 1.6, 1.6) [dot-dot-dash-dashed black], (1.3 GeV, 1.5, 1.5) [dashed blue], (1.4 GeV, 1, 1) [dot-dashed magenta], and (1.5GeV, 0.8, 0.8) [dotted cyan]. The $c\overline{c}$ total cross sections are shown on the right-hand side.

Calculations of $b\bar{b}$ and Υ Better Behaved

Bottom quark mass is large enough for K factors to be smaller and $b\bar{b}$ cross section more reliable
FONLL mass and scale choices work well in this case

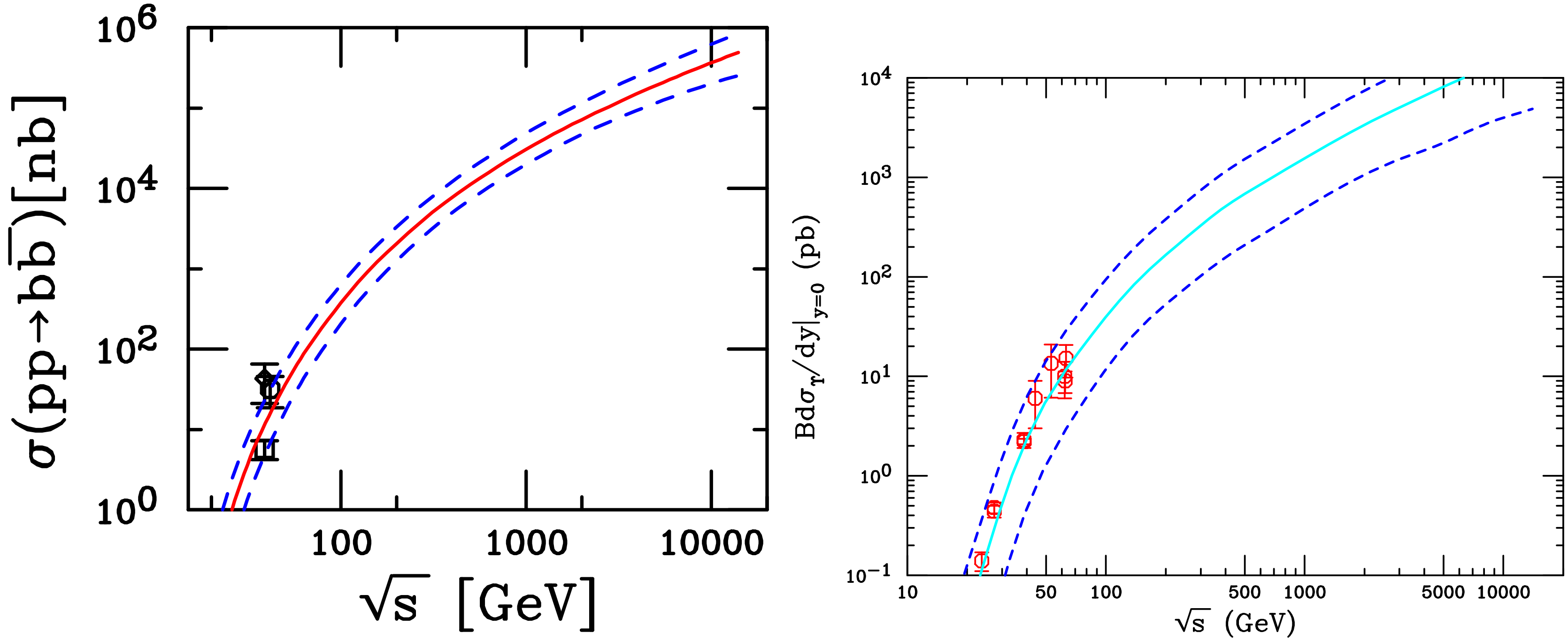

Figure 9: **The $b\bar{b}$ FONLL uncertainty band (left) and the combined Υ S states in the dilepton channel (right). Both are calculated to NLO in the CEM. [After Phys. Rept. 458 (2008) 1.]**

CEM p_T Distributions

Without intrinsic k_T smearing (or resummation) the $Q\overline{Q}$ p_T distribution (LO at $\mathcal{O}(\alpha_s^3)$ while total cross section is NLO at this order) is too peaked at $p_T \to 0$, needs broadening at low p_T

Implemented by Gaussian k_T smearing, $\langle k_T^2 \rangle_p = 1$ GeV^2 for fixed target pp and πp, broadened for pA and AA, NLO code adds in final state:

$$g_p(k_T) = \frac{1}{\pi \langle k_T^2 \rangle_p} \exp(-k_T^2/\langle k_T^2 \rangle_p)$$

Broadening should increase with energy we make a simple linear extrapolation to obtain

$$\langle k_T^2 \rangle_p = 1 + \frac{1}{3n} \ln\left(\frac{\sqrt{s}}{\sqrt{s_0}}\right) \text{ GeV}^2$$

We find $n \sim 4$ agrees best with RHIC data

Note that unlike FONLL-like calculation of single inclusive heavy flavor with resummed logs of p_T/m, at large p_T distribution may be harder than it should be

CEM Comparison to RHIC pp J/ψ Data

CEM calculation reproduces shape of J/ψ p_T and y distributions rather well considering that normalization is set from RHIC energies and below with only one parameter

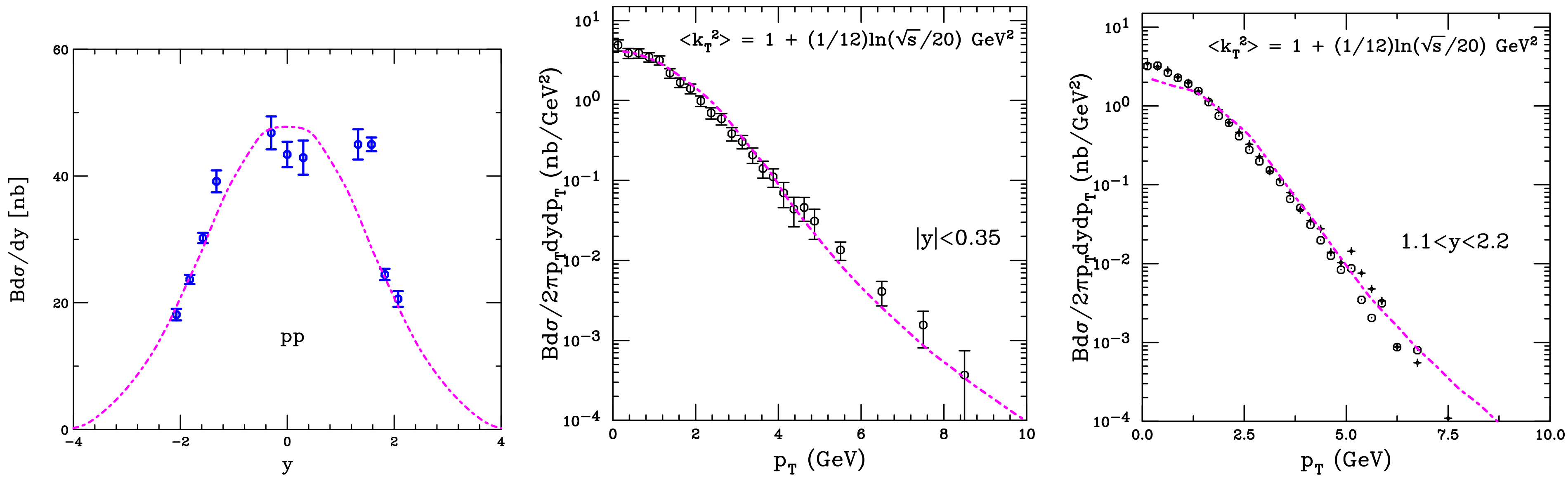

Figure 10: **PHENIX pp measurements compared to CEM calculation at $\sqrt{s} = 200$ GeV. The J/ψ rapidity distribution (left) and transverse momentum distributions at midrapidity (center) and in the muon arms (right). The results are calculated with CTEQ6M, $(m, \mu_F/m_T, \mu_R/m_T) = (1.2, 2, 2)$, $\langle k_T^2 \rangle = 1.38$ GeV^2. The forward result is scaled up by a factor of ≈ 1.4.**

CEM Comparison to Preliminary LHC pp Quarkonium Data

CEM calculation reproduces shape of J/ψ and $\Upsilon(1S)$ p_T distributions using CTEQ6M with $(m, \mu_F/m_T, \mu_R/m_T) = (1.2\ \mathrm{GeV}, 2, 2)$, $\langle k_T^2 \rangle = 1.38$ GeV^2 and $(m, \mu_F/m_T, \mu_R/m_T) = (4.75\ \mathrm{GeV}, 1, 1)$

No additional scale factor included

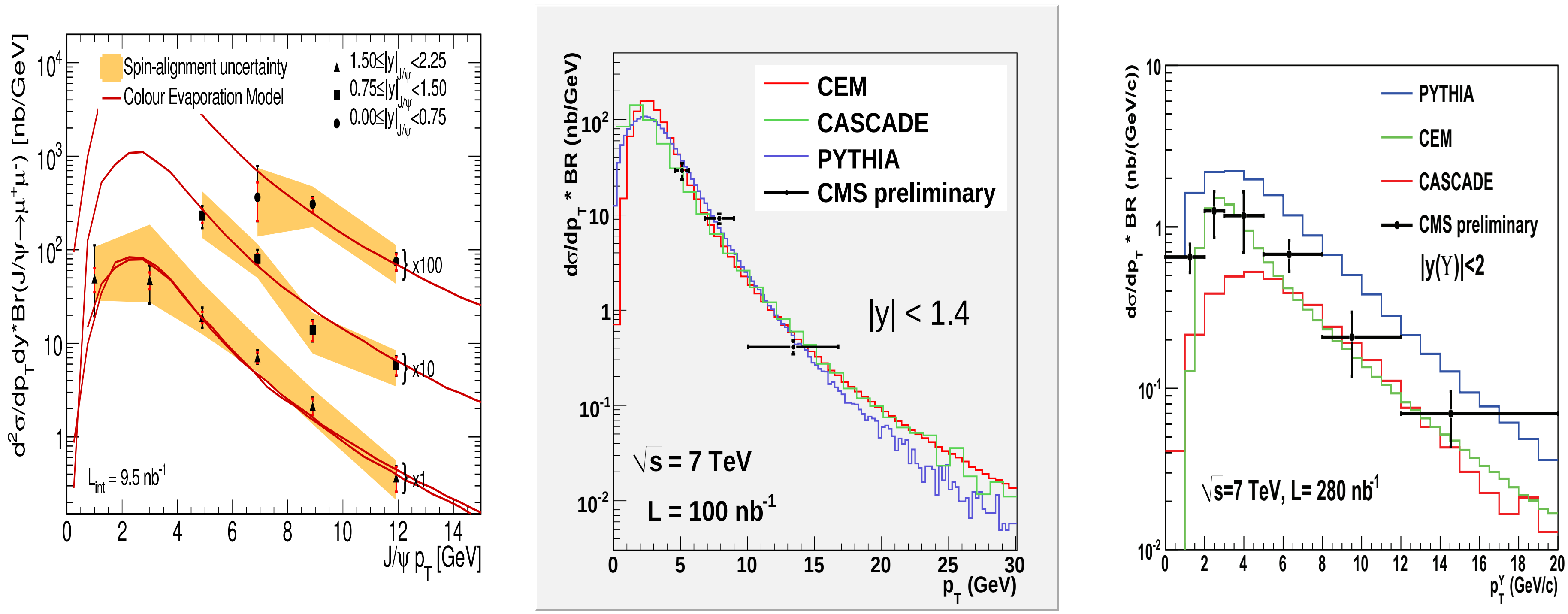

Figure 11: **ATLAS (left) and CMS (middle) J/ψ and CMS $\Upsilon(1S)$ (right) cross sections at 7 TeV compared to CEM calculations.**

CEM Uncertainty Using $c\overline{c}$ Fits

We show results both with $\mu_F = m_T$ and $2m_T$ even though $2m_T$ is clearly more consistent with overall energy dependence of cross section

For a given factorization scale curves have same slope, as expected

Normalization is fixed from individual fits

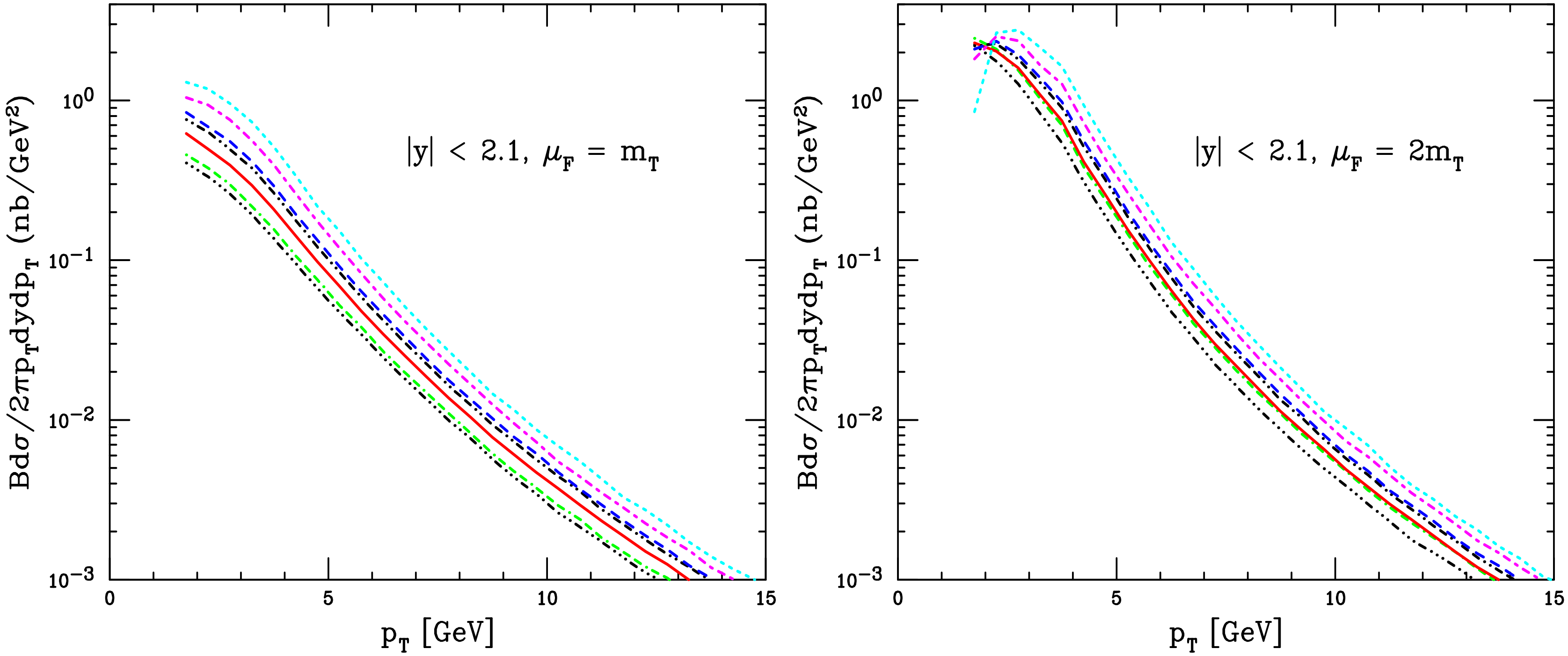

Figure 12: **The prompt J/ψ p_T distributions in the rapidity interval $|y| < 2.1$. The curves on the left-hand side are calculated with** $(1.1\,\mathrm{GeV}, 1, 3.9)$ **[dot-dot-dot-dashed black],** $(1.15\,\mathrm{GeV}, 1, 3)$ **[dot-dash-dash-dashed green],** $(1.2\,\mathrm{GeV}, 2, 2.3)$ **[solid red],** $(1.27\,\mathrm{GeV}, 1, 1.7)$ **[dot-dot-dash-dashed black],** $(1.3\,\mathrm{GeV}, 1, 1.5)$ **[dashed blue],** $(1.4\,\mathrm{GeV}, 1, 1)$ **[dot-dashed magenta], and** $(1.5\mathrm{GeV}, 1, 0.8)$ **[dotted cyan] while those on the right-hand side are with** $(1.1\,\mathrm{GeV}, 2, 4)$ **[dot-dot-dot-dashed black],** $(1.15\,\mathrm{GeV}, 2, 3)$ **[dot-dash-dash-dashed green],** $(1.2\,\mathrm{GeV}, 2, 2.2)$ **[solid red],** $(1.27\,\mathrm{GeV}, 2, 1.6)$ **[dot-dot-dash-dashed black],** $(1.3\,\mathrm{GeV}, 2, 1.4)$ **[dashed blue],** $(1.4\,\mathrm{GeV}, 2, 1)$ **[dot-dashed magenta], and** $(1.5\mathrm{GeV}, 2, 0.7)$ **[dotted cyan].** $(1.2\ \mathrm{GeV}, 2, 2)$ **[solid red],** $(1.4\ \mathrm{GeV}, 2, 1)$ **[dot-dashed magenta], and** $(1.5\ \mathrm{GeV}, 2, 0.7)$ **[dotted cyan] using the CT10 PDFs. The curves are normalized by the forward cross section fits at fixed-target energy.**

CEM Uncertainty Band for Υ

Wide uncertainty range in p_T distribution of Υ from FONLL choice of mass and scales

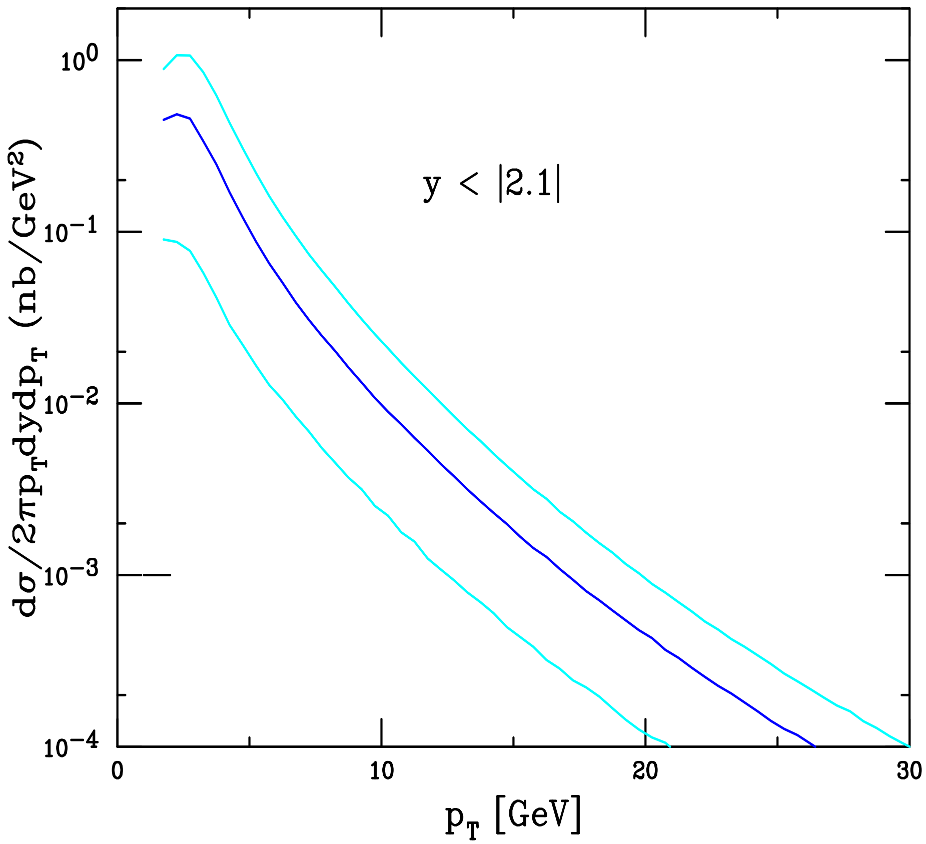

Figure 13: **The midrapidity Υ results using the FONLL uncertainty range. The blue curve is the central result while the cyan curves represent the upper and lower limits. The normalization is fixed for the central result.**

Summary .

- **Original version of CEM with exclusive NLO $Q\overline{Q}$ calculation does well against production data, especially considering that it has only one parameter to fix**
- **Attempt here to place some uncertainty on results for J/ψ and Υ**
- **No calculation of polarization yet available ($\neq$ no polarization) $\cdots$**